IDENTIFICATION OF DEFOCUS BLUR PARAMETERS AND
RESTORATION OF DEGRADED IMAGES*

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ABSTRACT

The problem of restoration of images blurred by uniform defocus is important in many applications. The solution proposed here identifies important parameters with which to characterize the point spread function (PSF) of the blur, given only the blurred image itself. We take advantage of the feature that there are a series of concentric circles in the Fourier spectrum domain of the defocused image, and the distance between two neighbor circles is in inverse proportion to the defocus radius R. In order to estimate the distance between circles, we use a coordinate transform to map the spectrum image into a new Cartesian coordinate system, in which the concentric circles turn out to be parallel lines. After detecting the distance between two neighbor lines, the blur parameter can be obtained according to the relationship of inverse proportion. When the PSF is estimated, a fast Kalman filter is adopted to restore the given image.

KEYWORDS: Defocus Blur; Parameter Estimation; Image Restoration; Kalman Filter

1. INTRODUCTION

It is inevitable that the image will be somewhat degraded during the process of generating, recording and transferring. In general, this degradation is very complicated, but in many situations, it is modeled as a linear spatially invariant process, which can be represented by the following discrete two-dimensional convolution [1]:

\[ g(x, y) = h(x, y) \ast f(x, y) + n(x, y) \]

where \( g(x, y) \) is the blurred image, \( f(x, y) \) is the true image, \( h(x, y) \) is the point spread function (PSF), \( n(x, y) \) represents the additive noise.

Defocus blur is caused by inaccurate focus in imaging systems like cameras, microscopes or other optical equipments. These situations exist in remote sensing, cosmos detecting, traffic control, medical treatment, analysis of micro phenomenon and many other important fields of research or application. Therefore restoration of defocused image has a theoretical significance as well as a prospect of applications.

Many existing image restoration algorithms assume that the PSF is known, but in practical it is not always the case. The restoration without knowing the PSF is called blind image restoration. There are mainly two kinds of algorithms of blind image restoration: one is combining the PSF estimation with image restoration process; and the other handles the two procedures independently. In this paper, our solution belongs to the latter. For an image degraded by the uniform defocus blur, first we developed a new method to identify its blur parameters by using the information of the PSF, true image and noise statistics [2]; and then apply a fast Kalman filter method [3] to finish the restoration process.

Kalman filter is an optimal recursive filter algorithm to the discrete-data linear filtering problem. It does not require all previous data to be kept in storage and reprocessed every time a new measurement is taken. So Kalman filter can save computation and storage requirements in a large amount. And in the past, considerable attention has been devoted to the application of the recursive Kalman filter to restore images degraded by both blur and additive noise [4][5].

2. PSF IDENTIFICATION

Early research results [6] show that the spectrum of the point spread function makes the spectrum of arbitrary direction motion-blurred image turn out to be a series of parallel bands which are perpendicular to the motion direction; the distance between spectrum center and the nearest dark line is in inverse proportion to the blur extend parameter of the motion-blurred image. So that the blur extend parameter can be identified in the frequency domain. As for defocus blur, PSF is modeled as a uniform intensity distribution within a circular disk:

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where disk radius $R$ is the only unknown parameter for this type of blur. Very similar to the motion-blur case, we found that in the Fourier spectrum of the defocused image there is a series of concentric circles, of which radii are in an arithmetical progression and the common difference is in inverse proportion to the defocus radius $R$.

An image is a set of pixels, which are regularly arranged in rows and columns. In this paper, we assume that pixels are square units and a discrete Cartesian coordinate system (denoted $C_1$ later, Fig. 1) is used to represent an image:

\[
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & \cdots & \cdots & i \\
2 & 3 & 4 & \cdots & \cdots & \cdots & j \\
3 & 4 & \cdots & \cdots & \cdots & \cdots & \cdots \\
4 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{array}
\]

Fig. 1. Discrete Cartesian coordinate system

For an $M \times N$ image degraded by defocus of unknown radius $R$, 2-D Discrete Fourier Transform (DFT) is applied to get the spectrum, of which the size is also $M \times N$. A polar coordinate system with the central point of the spectrum image $(M/2, N/2)$ chosen as its origin is established in the frequency domain to represent the spectrum image in another way (Fig. 2).

\[
\begin{array}{cccccc}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \circ & \circ & \circ & \circ & \circ \\
\bullet & \circ & \circ & \circ & \circ & \circ \\
\bullet & \circ & \circ & \circ & \circ & \circ \\
\end{array}
\]

Fig. 2. Polar coordinate system

In this coordinate system, a point is denoted by $(\rho, \theta)$, where $\rho$ is the radial distance from the origin and $\theta$ is the counterclockwise angle from the x-axis. We divide a perigon into $P$ equal angle parts, each of which counts $2\pi / P$ radians and this angle is used as the unit angle, and the unit of coordinate system $C_1$ is used as the unit length. So far, we could introduce a new discrete Cartesian coordinate system (denoted $C_2$ later), and for a pixel $(i, j)$ in this new coordinate system, $i$ counts the unit angles contained in $\theta$ while $j$ counts the unit lengths contained in $\rho$, and this can be formulated by:

\[
\begin{align*}
i &= \left\lfloor \frac{\theta}{2\pi / P} \right\rfloor, & i=1, \ldots, P-1 \\
j &= \lfloor \rho \rfloor, & j=1, \ldots, \min(M/2, N/2)
\end{align*}
\]

where $\left\lfloor \cdot \right\rfloor$ round to the floor integer.

By adding an offset $(-\rho \sin \theta, \rho \cos \theta)$ to the origin point $(M/2, N/2)$, a new point $(x, y)$ is obtained in coordinate system $C_1$. As shown in Fig. 3, around $(x, y)$ there are four pixels, and we choose intensity value of the one which minimizes the Euclidean Distance from its central point to $(x, y)$ as that of the pixel $(i, j)$ from coordinate system $C_2$.

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & \cdots & \cdots \\
2 & 3 & 4 & \cdots & \cdots & \cdots \\
3 & 4 & \cdots & \cdots & \cdots & \cdots \\
4 & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{array}
\]

Fig. 3. Four pixels around $(x, y)$

As soon as intensity values of all pixels in coordinates system $C_2$ are determined, we get a new spectrum image in which the concentric circles turn out to be parallel lines. Very similar to that of the parallel lines in Fourier spectrum of motion-blurred image, distance between two neighbor lines and the defocus radius $R$ is inversely proportional. And after defocus radius $R$ is obtained, we could get PSF of the blurred image by employing formula (2).

3.KALMANFILTERANDITSAPPLICATION ONTOIMAGERESTORATION

3.1 Kalman filter theory

The Kalman filter addresses the general problem of trying to estimate the state $x \in \mathbb{R}^n$ of a discrete-time controlled process that is governed by the linear stochastic difference equation [7]

\[
x_k = Ax_{k-1} + Bu_k + w_{k-1},
\]

with a measurement $z \in \mathbb{R}^m$ that is

\[
z_k = Hx_k + v_k.
\]

The $n \times n$ matrix $A$ in the difference equation (5)
relates the state at the previous time step \( k-1 \) to the state at the current step \( k \), in the absence of either a driving function or process noise. The \( n \times l \) matrix \( B \) relates the optional control input \( u \in \mathbb{R}^l \) to the state \( x \). The \( m \times n \) matrix \( H \) in the measurement equation (6) relates the state to the measurement \( z_k \). The random variables \( w_k \) and \( v_k \) represent the process and measurement noise.

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations, and they are presented in the following forms.

**Time update equations:**

\[ \dot{x}_k = A \hat{x}_{k-1} + Bu_k, \]  
\[ P_k = AP_{k-1}A^T + Q. \]  

**Measurement update equations:**

\[ K_k = P_k H^T (H P_k H^T + R)^{-1}, \]  
\[ \dot{\hat{x}}_k = \hat{x}_{k-1} + K_k(z_k - H\hat{x}_{k-1}), \]  
\[ P_k = (I - K_k H)P_k. \]

The time update equations are responsible for projecting forward the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations.

After each time and measurement update pair, the process is repeated with the previous a posteriori estimates used to project or predict the new a priori estimates. This recursive nature is one of the very appealing features of the Kalman filter: it makes practical implementations much more feasible than an implementation of a Wiener filter [8] which is designed to operate on all of the data directly for each estimate.

### 3.2 Application to image restoration

It is assumed that the original image can be presented by a zero-mean homogeneous \( N \times N \) discrete random field. The image can then be modeled by the semicausal type of model [3]:

\[ x(m,n) = \sum_{p,q \in W} a(p,q)x(m-p,n-q) + u(m,n), \]  

where

\[ W = \{p,q : 0 \leq p \leq p_s, 0 \leq q \leq q_s, (p,q) \neq (0,0)\}. \]

\[ x(m,n) \] is the intensity value of the pixel \((m,n)\) in the original image, and \( u(m,n) \) is the noise input term. \( W \) is the random field of the chosen model. All points on a line with index \( m \) may be combined into a vector \( X(m) = [x(m,1), \ldots, x(m,N)]^T \). By rewriting equations (12) and (13), we obtain a matrix-vector equation:

\[ A_p X(m) = -\sum_{p=1}^{p_s} A_p X(m-p)+U(m). \]  

Similarly, we could also rewrite the discrete two-dimensional convolution (1) of the image model into the following matrix-vector form:

\[ Y(m) = \sum_{k=1}^{k_s} H_k X(m-k) + N(m). \]

here, \( X(m) \) is the image vector \([x(m,1), \ldots, x(m,N)]^T\), and \( Y(m) \) is the observation vector \([y(m,1), \ldots, y(m,N)]^T\), \( m=1, \ldots, N \), and \( A_p \) and \( C_n \) are \( N \times N \) matrices of band-Toeplitz structure. We can make use of the property that a Toeplitz matrix can be approximated by a circulant matrix. A circulant matrix has the attractive property that it can easily be diagonalized by means of the discrete Fourier transform (DFT), and then equations (14) and (15) reduce to a set of decoupled equations. Based on this model a set of low-order Kalman filters are suitable for parallel processing of the data in the transform domain. The number of computations is reduced from an order of \( O(N^3) \) to an order \( O(N^2 \log_2 N) \).

### 4. RESULTS

The experiments are carried out by using the MATLAB Image Processing Toolbox [9]. The original image "aerial sight" in Fig.4(a) was artificially blurred by a uniform defocus PSF with \( R=6 \) and \( R=12 \) (Fig.4(b) and Fig.4(c)), and their spectrum images are shown respectively in Fig.4(d)-(f). New spectrum images in coordinate system \( C_2 \) (Fig.4(g) and Fig.4(h)) were gained by applying to Fig.4e and f the method described above. We can now obtain the defocus radius \( R \) and also the PSF of the two blurred images. At last, restoration results by a fast Kalman filter method are given in Fig.4(i) and Fig.4(j).

### 5. CONCLUSION

Blind image restoration can be done in two steps: first identify the PSF parameters of the blurred image, and then choose a proper algorithm to restore it by using the information from the previous step. Since the restoration process is based on the identification results, correct identification of the PSF parameters permits fast high resolution restoration of the blurred image. In this paper, a new method of identifying the defocus radius of uniform defocus blur is presented. For a defocused image with unknown defocus radius, first, two-dimensional DFT is used to get its Fourier spectrum. Second, by transforming the spectrum image from coordinate system \( C_1 \) to \( C_2 \), we obtain a new spectrum image in \( C_2 \), in which parallel lines of
equidistance can be found. And then we determine defocus radius $R$ according to the fact that it is in inverse proportion to the distance between two neighbor lines.

The restoration of uniform defocus blurred images can be found in many applications, it is believed that our method has a good prospect in future researches.

REFERENCES