

## Traffic dynamics in scale-free networks with limited buffers and decongestion strategy

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**Abstract.** We studied the information traffic in Barabási–Albert scale-free networks wherein each node has a finite queue length to store the packets. It is found that in the case of the shortest path routing strategy, the networks undergo a first-order phase transition, i.e. from a free flow state to a full congestion state, with increasing packet generation rate. We also incorporate the random effect (namely random selection of a neighbor to deliver packets) as well as a control method (namely the packet-dropping strategy of the congested nodes after some delay time  $T$ ) into the routing protocol to test the traffic capacity of the heterogeneous networks. It is shown that there exists an optimal value of  $T$  for the networks to achieve the best handling ability, and the presence of an appropriate random effect also contributes to the performance of the networks.

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**1. Introduction**

In the last few years, complex networked systems have attracted much attention, ranging from the areas of physics, sociology and biology to technology and many others [1]–[5]. It has been well proved that the topological features of underlying interaction networks have great impact on the final outcomes of the dynamics taking place on them [1]–[5]. For example, the scale-free topology of a network results in a vanishing threshold of epidemic spreading on it with increasing network size [6], gives rise to a robust behavior against random failures and is fragile for aimed attack [7]; the networks with a homogeneous degree distribution, a small average path length and a small clustering coefficient are found to more easily retain synchronization [8].

Spreading and transport problems (epidemics, opinion, cascading, information, etc) have received much attention from researchers and they have been investigated in various kinds of complex networks [2, 3, 5], [9]–[12]. In this paper, we focus on information traffic in networks (modeling the Internet), since it plays a more and more important role in our daily life [13], e.g. data resources, e-business, online games, and many others. The ever-increasing number of users of the Internet and hence the need for a tremendous amount of information transport make it necessary to study how the topological properties of the underlying infrastructures influence the traffic flow [14]. In studies of the information traffic in complex networks, the following two problems are mostly focused on [15]–[22]: given the underlying network, what is the efficient routing protocol to optimize the packet delivery, and given a routing protocol, what type of structure of the network is optimal, i.e. the performance of the structure or evaluation of the effect of the structure on the traffic. (Optimality is defined as the minimization of the average packet arrival time and the maximization of the packet-handling capacity of the network.) In this paper, we would like to investigate in detail how the underlying topological structure affects the data traffic in complex heterogeneous networks.

All too often, the buffer size of the nodes is assumed to be infinite, i.e. all the nodes can receive as many data packets as possible [15]–[24]. However, due to physical constraints, we know that all data-processing machines have a finite buffer (or queue length, denoted by  $L$  in the rest of this paper for simplicity) to store data packets. It is reasonable and necessary to take into account this fact in traffic studies. So, in the present paper, we discard the infinite buffer assumption, but rather consider a finite buffer of each node, and study how this restraint affects the data traffic. In addition, we also incorporate a random effect as well as a control method into the routing protocol to study the performance of the networks. It is shown that due to the finite storage of the nodes, a first-order phase transition emerges in the information transportation process. The random effect and the control method in the routing protocol are also found to have a great influence on the handling capacity of the networks.

## 2. The model

It has been proposed that scale-free network topology is a suitable candidate for the structure of the Internet at the autonomous system level [25, 26]. For simplicity, we use the well-known Barabási–Albert (BA) scale-free network model [27] as the physical infrastructure on top of which a packet delivery process is taking place. The BA model contains two generic mechanisms of many real complex systems: growth and preferential attachment [1, 27], which can be constructed as follows. Starting from  $m_0$  nodes, one node with  $m$  links is attached at each time step in such a way that the probability  $\prod_i$  of being connected to the existing node  $i$  is proportional to the degree  $k_i$  of that node, i.e.  $\prod_i = k_i / \sum_j k_j$ , where  $j$  runs over all existing nodes. In the present work, the total network size is fixed as  $N = 1000$  and the parameters are set to be  $m_0 = m = 3$  (hence the average connectivity of the network is  $\langle k \rangle = 6$  [1]). The degree distribution of the generated BA network  $P(k)$ , which denotes the probability of a randomly selected node in the network having exactly degree  $k$ , follows a power law  $P(k) \sim k^{-\gamma}$  with the exponent  $\gamma = 3$  in the large degree limit.

Each node of the underlying infrastructure plays the roles of the host and a router at the same time. Due to the finite storage capacity of the nodes, we now define that a node is congested if its buffer is fully filled by packets. The packet transmission on the network, i.e. the packet-generating process and the packet delivery process, is implemented by a discrete time parallel update algorithm. At each time step, the probability for node  $i$  to generate a packet is  $R$  if there is some free space in its buffer (i.e. available in its queue); otherwise no new packet is inserted. Once a packet is created, its destination is chosen uniformly at random among the other  $N - 1$  nodes of the network. Each newly inserted packet is placed at the end of the queue of the node which generates it. After all the nodes have finished the packet-generating process, they start to deliver the packets stored in their queue, which are composed of all packets that were sent to them by their neighbors in the previous steps and packets created by themselves (if any). For simplicity, and without loss of generality, we assume that all nodes have the same processing capacity of one data packet per time step. The first packet in the queue is then sent to a neighbor following some routing protocol.

Although in modeling communication networks like the Internet, the routing process following the shortest path (SP) from a given source to its destination is usually preferable, many previous studies have shown that a certain degree of stochasticity can, to some extent, enhance the traffic-handling capacity of the underlying network [28]–[31]. On the other hand, it has been suggested that for the Internet almost 70% of data packets are forwarded along their SP from the source to the destination [32]. Inspired by these two factors, we introduce in our model some randomness for the packet delivery process, namely, we let data packets be delivered along their SP from the source to the destination with a probability  $(1 - p)$ ,<sup>2</sup> and be sent to a random neighbor with probability  $p$ . For this type of randomness in packet routing, we want to remark that in the realistic case, routing paths, when not along the SP, usually go preferentially to a set of nodes which are landmarks [33]. However, it is inconvenient to determine which nodes are landmarks in a model network, and for the sake of simplicity we just let all the neighbors have the same chance. Whenever a packet is forwarded along the SP, if there exist several shortest paths, we just select randomly one of them to deliver the packet. For  $p = 0.0$ , we recover the SP delivery protocol. For other values of  $p$  greater than zero, we incorporate a random effect

<sup>2</sup> Here, we simply define the SPs of any given two nodes as the paths with minimal number of hops when going from one to the other, whereas this is not always the actual case in realistic situations.

into the packet delivery. To make a close correlation to the realistic scenario of data traffic, here we restrict our studies to values of  $p$  between 0.0 and 0.2. Once a packet reaches its destination through the above routing protocol, it will be removed from the system.

We want to remark that the packet generation rate  $R$  in our model is different from that in the case of the infinite queue length [16]–[22]. In fact, since the congested nodes (whose buffers are fully filled) temporarily cannot generate any new data packets, the *effective* average packet generation rate of the whole network is equal to or less than  $R$ . Nevertheless, we also denote the average packet generation rate by  $R$  for convenience.

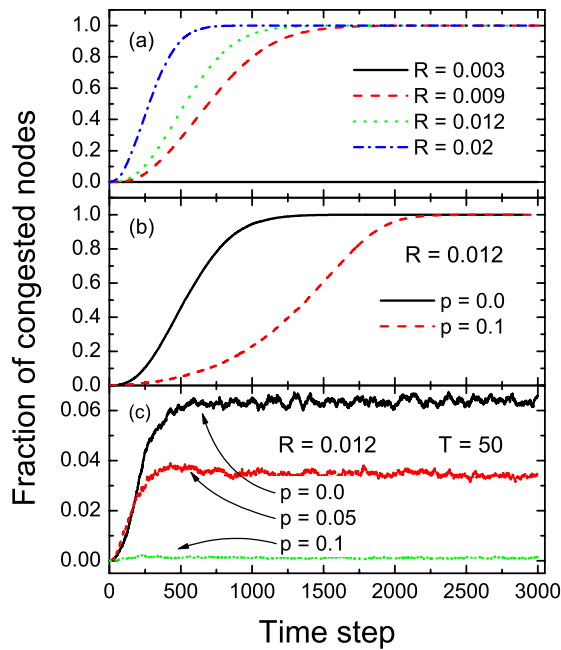
### 3. Results and discussion

According to the above algorithm for packet delivery, one can expect that for large values of  $R$ , the whole network would fall easily into a fully congested state due to the finite queue length of the nodes. In figure 1(a), we show four typical time evolution series of the average fraction of congested nodes in the BA scale-free networks with total size  $N = 1000$ ,  $\langle k \rangle = 6$  and  $L = 5$ .<sup>3</sup> The different styled lines correspond to different packet generation rates  $R = 0.003, 0.009, 0.012$  and  $0.02$ , respectively. From figure 1(a), one can see that for sufficiently small  $R$ , e.g.  $R = 0.003$ , the networks sustain a free flow state (absence of congestion), while for large values of  $R$ , the networks are doomed to jam and the fraction of congested nodes in the networks increases very fast with the increment of time steps. The greater the value of  $R$  is, the more sharply the curves ascend. We have also implemented simulations for other values of  $N$ ,  $\langle k \rangle$  and  $L$ . The qualitative behaviors of the time evolution curves shown in figure 1 do not change (results not shown here).

In figure 1(b), the time evolution of the average fraction of congested nodes is plotted for a special value of  $R = 0.012$  with the incorporation of a random effect in the routing protocols ( $p = 0.1$ ). For comparison, the result for  $p = 0.0$  is also shown. It is clear that the curve for  $p = 0.1$  ascends more slowly than that for  $p = 0.0$ , which indicates that some appropriate degree of stochasticity for routing protocols can improve the handling ability of the network in accordance with previous observations in the case of infinite queue length [28]–[31]. Despite this point, we can see that the network is still doomed to jam in the long time limit, which is also caused by the finite buffer size of the nodes. (Figure 1(c) will be discussed later.)

Since whether the underlying networks are congested or not depends strongly on the packet generation rate  $R$ , we would like to first investigate the behavior of the congestion as a function of  $R$ . In the inset of figure 2, we show the results of a typical sample of the information traffic in a BA network. The packets are forwarded by using the SP routing protocol. It is obvious that a first-order phase transition emerges at a certain value of  $R_C$  above which the network is doomed to congestion. Due to the finite size of the considered BA network ( $N = 1000$  in the present case), the value of  $R_C$  for the traffic in different network realizations will vary to some extent. To obtain a more precise picture of the phase diagram, we have implemented 15 000 samples including a set of 50 realizations of the BA network, and 300 independent experiments on each of them. The total simulation time is  $10^4$ ; we have checked that for the currently studied cases ( $N = 1000$  and  $L = 5$ ), this sample time is enough for the systems to attain final equilibrium. We calculated the total number of times that the networks fall into full congestion by varying

<sup>3</sup> Other selection of finite value of  $L$  does not change the qualitative behavior of the results shown in the text. However, a much longer simulation time is needed for the systems to attain equilibrium for large  $L$ .



**Figure 1.** Time evolution of the congested nodes in BA scale-free networks of size  $N = 1000$  with  $m = m_0 = 3$ . (a) Data packets forwarded by using the SP routing protocol. (b) The case of some random effect involved in the packet delivery process. (c) The case of the packet-dropping strategy applied to the congested nodes. (See the text for details.)

the value of  $R$ , and then the results are normalized by the total number of samples. The final results, i.e. the average probability  $f_C$  of congestion as a function of  $R$ , are plotted in the main panel of figure 2(a), and the threshold  $R_C$  is about 0.005(5) in the present case (obviously, for other values of  $N$ ,  $m_0$  and  $L$ , this value will vary<sup>4</sup>).

Let us give an approximate analytical estimation for the critical packet generation rate  $R_C$  above which the congestion phase occurs. According to the queuing theory [34], in the free flow state we know that the average queue length  $\langle l_i \rangle$  is given (assuming unity processing capacity at each time step) by

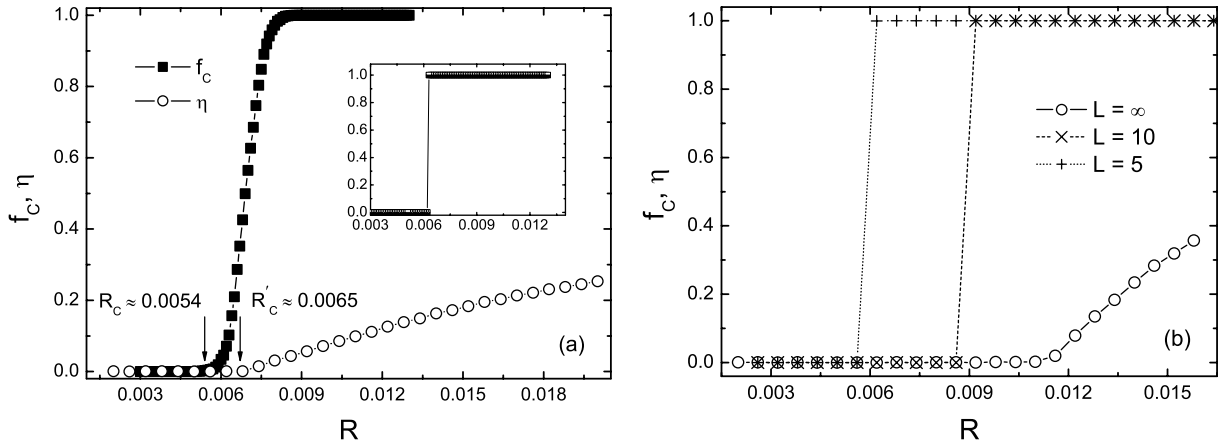
$$\langle l_i \rangle = \frac{c_i}{1 - c_i}, \quad (1)$$

where  $c_i$  is the number of packets passing through the node  $i$ , and scales its betweenness  $b_i$  as [16, 28, 34]:

$$c_i = \frac{Rb_i}{N - 1}. \quad (2)$$

In the case of finite buffer size, if any  $l_i$  exceeds the maximum size  $L$  (i.e. congested), this will cause its neighbors to be easily congested in sequential time steps. The congestion will cumulate continuously in the system, and the whole network will definitely get jammed sooner or later.

<sup>4</sup> In our present studies, we are concerned about the value of  $R$  below which all the sampling systems are in free flow state, and we call this value ' $R_C$ '.

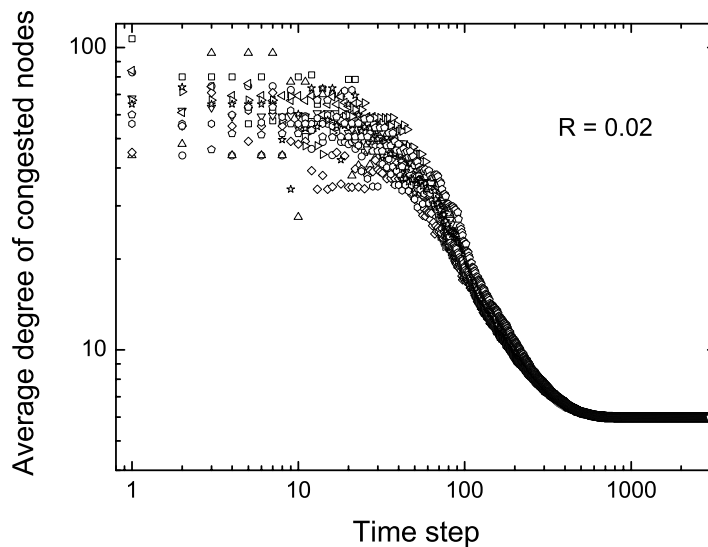


**Figure 2.** (a) Probability of congestion  $f_C$  versus the data packet generation rate  $R$  in BA networks of size  $N = 1000$  with  $m = m_0 = 3$ . Each data point is obtained by averaging over 15 000 samples (a set of 50 realizations of the BA network, and 300 independent experiments for each of them). The inset shows the results of a typical experiment of information traffic in a BA network. The Y-axis of the inset denotes the fraction of congested nodes. (b) Fraction of congested nodes as a function of  $R$  in a regular ring network (RRN) for several values of  $L$ . For comparison, the congestion level  $\eta$  of both the BA and RRN networks in the case of infinite buffer size is also given by open circle symbols.

Thus, we can estimate the transition point by setting the average queue length of the node with maximum betweenness to be  $L$ . After algebraic calculations, we have

$$R_C = \frac{L(N-1)}{(L+1)b_{\max}}, \quad (3)$$

where  $b_{\max}$  is the largest betweenness of any node in the network. For infinite buffer size, the underlying network will undergo a second-order phase transition from free flow to congestion with the increase of  $R$  [15]. The transition point is  $R'_C = (N-1)/b_{\max}$  (greater than  $R_C$  for finite buffer size), and the congestion level is characterized by  $\eta = \lim_{t \rightarrow \infty} \frac{\langle \Delta N \rangle}{NR \Delta t}$ , where  $\Delta N = N(t + \Delta t) - N(t)$ ,  $N(t)$  is the total number of packets in the network at time  $t$  and  $\langle \cdot \cdot \rangle$  indicates the average over time windows of  $\Delta t$  [15, 29]. In figure 2(a), we also give the variation of  $\eta$  as  $R$  increases by open circles. It is indeed clear that the critical value of  $R_C$  for finite buffer size is smaller than that for the infinite case. Here, we want to remark that equation (3) is an overestimation of  $R_C$ , since  $\langle l \rangle$  is the average queue length of the nodes. The fluctuation of the queue length of the nodes will obviously suppress the critical value of  $R$ . For the sake of comparison and verification of the above statement, we have also checked the traffic dynamics on a regular ring network (RRN) with finite buffer size. The total size and average connectivity of the RRN are kept the same as those of the BA networks studied above. The results are presented in figure 2(b). For the case of infinite buffer size  $L = \infty$ , the critical value  $R_C$  is approximately  $R_C \approx 2\langle k \rangle / N = 2 \times 6 / 1000 = 0.012$  [29]. When finite buffer size is taken into account, the critical point is shifted towards zero. A smaller  $L$  value gives rise to a smaller value of  $R_C$ . In particular, the phase transition from the free flow state to the congestion state is also found to be of first order for finite values of  $L$ .



**Figure 3.** Time behavior of the average degree of the congested nodes for congestion outbreaks ( $R = 0.02 > R_C$ ) in BA networks of size  $N = 1000$  with  $m = m_0 = 3$ . The shown results are for ten independent samples (plotted by different symbols).

From figure 1(a), we can see that, once some nodes are jammed, the congestion spreads very fast in the whole network if no control measure is deployed. From the point of view of developing control strategies to prevent the whole network from malfunction, it is valuable to figure out the detailed knowledge of the way the congestion spreads through the network. Since the packets are delivered according to the SP protocol, the hubs (having large degrees and hence high betweenness) are more prone to congestion than those nodes with small degrees. A simple way of characterizing the congestion diffusion through the network is to study a convenient quantity, namely the average degree of congested nodes (if any), in numerical transporting experiments. We show in figure 3 this quantity for ten independent samples in the BA networks as a function of simulation time step. It is clear that the curves show an initial plateau whose values are far greater than the average connectivity of network 6, which indicates, undoubtedly, that the large-degree nodes in the network are preferentially congested. After the hubs are blocked, the remaining nodes with the largest degree are then congested, which is reflected by the smooth decrease of the curves. Taken together, the dynamical spreading process of congestion is clear: with more and more packets inserted into the network, the hubs with high betweenness are congested first, and then the congestion spreads, going towards those nodes with smaller and smaller degrees. Finally, the whole network is totally jammed, and the average degree of the congested nodes is the same as the average connectivity of the BA networks.

From the above scenario of the congestion propagation in heterogeneous networks, one may realize that an efficient way to prevent the whole network from malfunction is to detect the blocked nodes in the very early stage of congestion, and to deal with the problem as soon as possible. However, due to the fluctuation of information flow in real communication systems [35] as well as many other physical restraints, it is inconvenient and even impossible to make immediate measurements and treatments on the congested nodes before a large-scale breakdown of the system. It is also proposed that the presence of assortative links among those

highly connected hubs can evidently improve the efficiency of the underlying network [24]. However, since most communication networks can be regarded as embedded in a geographic surface, it is very costly to connect hubs that are far apart. It is easier to let the congested nodes themselves have the ability to get out of the jam in terms of some appropriate instructions. As an alternative way, in the following part we implement a packet-dropping strategy for the congested nodes to prevent the whole network from jamming.

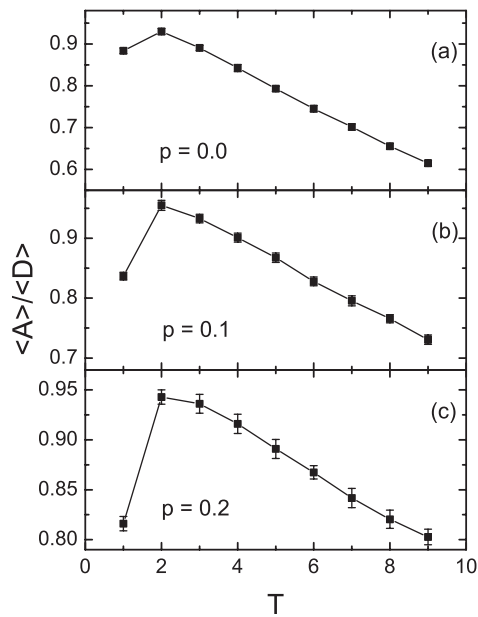
We assume that if a node is congested and the persistence of this congested state exceeds some time  $T$ , then the node will simply empty its buffer to regain capacity for receiving and handling packets. In practice, this method can be easily implemented by using control software, for example, the long time congested router can call for a pre-installed instruction to empty its buffer. In our computer simulations, if the congested state of a node lasts for more than  $T$  time steps (for convenience, the time that a node is just congested is marked by  $T = 1$ ), we reset its buffer size back to  $L = 5$ , and those stored packets in its buffer are discarded. Obviously, by doing so, we face the problem that many packets will be lost and will disappear forever in the network. On the other hand, however, if no response such as packet-dropping is implemented, the whole network would rapidly collapse. On balance, one acceptable method is to select an appropriate parameter  $T$  to achieve as little packet-loss as possible, while keeping the whole network functional.

In figure 1(c), the time evolution of the fraction of congested nodes for  $T = 50$  and several values of  $p$  are shown. The packet generation rate  $R$  is set to 0.012, which guarantees that the whole network is blocked without the implementation of the strategy of packet-dropping (as displayed in figure 1(b)). From figure 1(c), we note that even for so large a value of  $T$ , the networks remain at low-level congestion, and if some random effects are involved in the packet delivery, the congestion level may further decrease to a negligible level (the curve for  $p = 0.1$  in figure 1(c)). If  $T$  is very large, we know that the whole network is still likely to be jammed for large values of  $R$ ; if  $T$  is very small, however, many packets would be lost. Taking both into consideration, there should be an intermediate value of  $T$  to achieve optimal performance. To evaluate the efficiency of the packet-dropping method, we study the ratio of the average number of successfully arrived packets  $\langle A \rangle$  to the average number of dropped packets  $\langle D \rangle$  by tuning the values of the parameters  $T$ ,  $p$  and  $R$ .

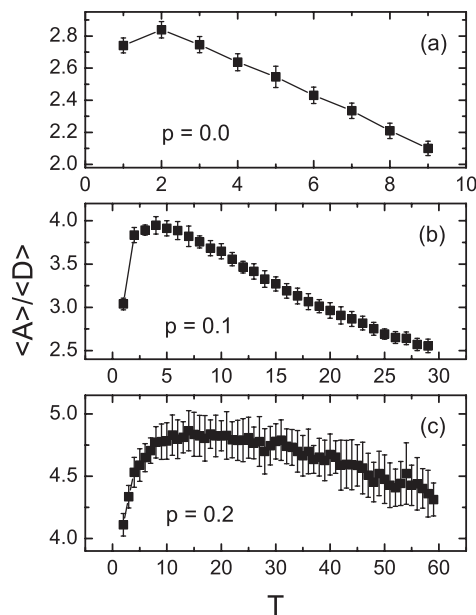
In figures 4–6, we show the quantity  $\langle A \rangle / \langle D \rangle$ , respectively, as a function of  $T$  for several values of  $p$  and  $R$ . The presented simulation results are obtained by averaging over 30 independent traffic realizations on BA scale-free networks of total size  $N = 1000$  and  $m = m_0 = 3$ . For a large packet generation rate  $R = 0.05$  (figure 4), there exists an optimal value for  $\langle A \rangle / \langle D \rangle$  at  $T = 2$ , which is insensitive to the detailed value of  $p$ . This result indicates that at times of high flux (large values of  $R$ ), the most efficient way to alleviate traffic congestion and sustain the overall traffic-handling ability of heterogeneous networks is to empty immediately the buffers of those nodes that are congested. However, it is worth pointing out that the simulation results in figure 4 clearly show that the most appropriate time for the congested nodes to empty their buffer is not the time that they are just jammed, but one more time step after they are congested. This perhaps is due to the high rate of packet loss induced by immediate dropping of the packets stored in the congested nodes' buffer, which otherwise may be delivered in the next time step to their unblocked neighboring nodes.

The above picture, however, is changed for smaller (yet greater than the  $R_C \approx 0.005$ ) values of  $R$  (figures 5 and 6). For  $p = 0$ , i.e. when the packets are forwarded by using the SP protocol, the most appropriate time for the congested nodes to empty their buffer is the same as



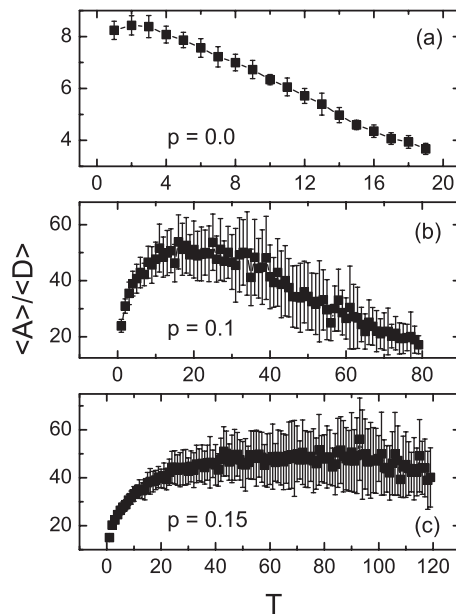


**Figure 4.** The ratio of the average number of successfully arrived packets,  $\langle A \rangle$ , to the average number of dropped packets,  $\langle D \rangle$ , as a function of the delay time  $T$  for different values of  $p$ . The error bar denotes the standard deviation from a total of 30 traffic simulations on one realization of the BA network. The parameter  $R$  is equal to 0.05.



**Figure 5.** As shown in figure 4, but for  $R = 0.02$ .

before: at  $T = 2$ , namely one more time step after they are congested (figures 5(a) and 6(a)). When there exist some stochastic effects in the packet delivery, i.e. for any value of  $p$  greater than 0, the most appropriate time for the congested nodes to empty their buffer shows nontrivial



**Figure 6.** As shown in figure 4, but for  $R = 0.012$ .

behavior, which depends closely on the values of both  $R$  and  $p$ . More precisely, the smaller the value of  $R$  as well as the larger the value of  $p$ , the larger the suitable time  $T$  is (figures 5(b) and (c) and figures 6(b) and (c)). This means that at times of low flux, in order to achieve high packet arrival rate, the right way is to just let the congested nodes remain in their blocked state for some appropriate time. It is worth stressing that the condition of  $p > 0$  should be satisfied, which is the actual case in realistic communication systems [32]. Finally, we want to remark that the curves shown in figures 5 and 6 are obtained from averages of 30 independent traffic experiments on one BA network realization. However, we have checked that for another independent realization of the underlying infrastructure, the shape of the curves may deviate to some extent, but all the qualitative properties of them (just as was shown in figures 5 and 6) remain absolutely unchanged.

#### 4. Conclusion

In summary, we have studied the information traffic in BA scale-free heterogeneous networks. The nodes are endowed with finite buffer size and the same capacity for processing packets in each time step. It was found that for a sufficiently small packet generation rate, the networks sustain a free flow state, whereas for a large packet generation rate, the underlying infrastructures sooner or later fall into a totally jammed state. The phase transition of the heterogeneous networks from the free flow state to the fully congested state with the increment of packet generation rate is of first order. Whenever the heterogeneous networks are going to be jammed, we have shown that the congestion takes first control of the hub nodes in the networks, and then it rapidly invades the whole network via a hierarchical cascade progressively through the nodes with smaller and smaller degrees. We have also investigated a control method, namely the packet-dropping strategy with a delay time  $T$ , to keep the whole network functional in the case of a high packet generation rate. It was found that by using this simple strategy, a network

can sustain well its packet-handling ability at the expense of lowering the packet arrival rate. The efficiency and quality of the control method are determined by some correlated factors, e.g. the magnitude of the packet generation rate and the degree of stochasticity of the routing protocols as well.

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