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Traffic dynamics in scale-free networks with limited packet-delivering capacity

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ABSTRACT

We propose a limited packet-delivering capacity model for traffic dynamics in scale-free networks. In this model, the total node's packet-delivering capacity is fixed, and the allocation of packet-delivering capacity on node i is proportional to k_i^{ϕ}, where k_i is the degree of node i and ϕ is a adjustable parameter. We have applied this model on the shortest path routing strategy as well as the local routing strategy, and found that there exists an optimal value of parameter ϕ leading to the maximal network capacity under both routing strategies. We provide some explanations for the emergence of optimal ϕ.

1. Introduction

Since the discovery of the small-world effect by Watts and Strogatz [1] and the scale-free property by Barabási and Albert [2], the structure and dynamics of complex networks have attracted growing interest and attention from the physics community [3–7]. Due to the increasing importance of large communication networks such as the Internet and WWW, information traffic on complex networks has drawn more and more attention [8–32]. The ultimate goal of studying these large communication networks is to control the traffic congestion and improve the efficiency of information transportation.

Researchers have proposed some models to mimic the traffic on complex networks by introducing packets generating rate R as well as randomly selected sources and destinations of each packet [9–13]. In these models, the capacity of networks is measured by a critical generating rate R_c. At this critical rate, a continuous phase transition from a free flow state to a congested state occurs. In the free-flow state, the numbers of created and delivered packets are balanced, leading to a steady state. While in the jammed state, the number of accumulated packets increases with time due to the limited delivering capacity or finite queue length of each node. It has been found that both network structure and packet routing strategy can influence the capacity and efficiency of information transportation.

The node packet-delivering capacity, that is, the number of packets a node can forward to other nodes in each time step, is assumed to be a constant or proportional to the node's degree in most previous works. Obviously more packet-delivering capacity can help to alleviate traffic congestion, but extending packet-delivering capacity will bring economic and technique

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pressure. So the question arises: how to rationally allocate the limited packet-delivering capacity onto nodes in order to maximize the network capacity? In the following, we will explore this question in scale-free networks.

The paper is organized as follows: In Section 2, the traffic model is introduced. The simulation results are presented and discussed in Section 3. The conclusion is given in Section 4.

2. The model

Recent studies indicate that many communication networks such as the Internet and WWW are heterogeneous with degree distribution following the power-law distribution $P(k) \sim k^{-\gamma}$. In this paper, we use the well-known Barabási–Albert (BA) scale-free network model [2] as the physical infrastructure to study information traffic flow. The BA model can be constructed as follows: starting from m_0 fully connected nodes, a new node with m edges is added to the existing graph at each time step according to preferential attachment, i.e., the probability Π_i of being connected to the existing node i is proportional to the degree k_i.

Once the network is generated, it remains fixed, and the traffic dynamics is modeled on top of it as follows: at each time step, there are R packets generated in the system, with randomly chosen sources and destinations. All the nodes act as both hosts and routers, and node i can deliver at most C_i packets per time step towards their destinations. Once a packet arrives at its destination, it will be removed from the system. The queue length of each node is assumed to be unlimited and the FIFO (first in first out) discipline is applied at each queue [9,10].

Packets can be delivered according to different routing strategies. In this paper, we consider the network traffic in the cases of both the shortest path and local routing strategy. The local routing strategy [17] can be described as follows. Each node performs a local search among its neighbors. If the packet's destination is found within the searched area, i.e., among the node's immediate neighbors, it is delivered directly to its target. Otherwise, it is forwarded to a neighbor node i, according to the probability:

$$\Pi_i = \frac{k_i^{\alpha}}{\sum_j k_j^{\alpha}}, \tag{1}$$

where the sum runs over the neighbors (searched area) of the searching node, k_i is the degree of node i and α is an adjustable parameter. The average packet-delivering capacity of the network is:

$$\langle C \rangle = \frac{1}{N} \sum_{i=1}^{N} C_i. \tag{2}$$

Based on economic and technique considerations, it's significant to investigate how to allocate packet-delivering capacity onto nodes when $\langle C \rangle$ is fixed. Since a scale-free network is heterogeneous, packet-delivering capacity can be allocated in the form of:

$$C_i = N \langle C \rangle \frac{k_i^{\phi}}{\sum_{j=1}^{N} k_j^{\phi}}, \tag{3}$$

where ϕ is an adjustable parameter. For $\phi > 0$ ($\phi < 0$), nodes with higher (smaller) degrees have larger packet-delivering capacity. When $\phi = 0$, all nodes have the same packet-delivering capacity. Noting that C_i may be an integer plus a fractional part, the fractional part is implemented as the probability of delivering additional packets in a time step.

3. Simulation results

In order to characterize the network capacity, we use the order parameter presented in Ref. [8]:

$$\eta(R) = \lim_{t \to \infty} \frac{1}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}, \tag{4}$$

where $\Delta N_p = N_p(t + \Delta t) - N_p(t)$, $\langle \cdots \rangle$ indicates the average over time windows of width Δt, and $N_p(t)$ represents the number of data packets within the network at time t. With increasing packet generation rate R, there will be a critical value of R_c that characterizes the traffic phase transition from free flow to a congested state. When $R < R_c$, $\langle \Delta N_p \rangle = 0$ and $\eta(R) = 0$, corresponding to the case of free-flow state. However, for $R > R_c$, $\eta(R)$ is a constant larger than zero, the packets will continuously pile up within the network and the system will collapse ultimately. Therefore R_c is the maximal generating rate under which the system can maintain its normal and efficient functioning. Thus the overall capacity of the system can be measured by R_c.

Fig. 1 reports the order parameter η versus generating rate R for different parameter ϕ under the shortest path routing strategy. One can see that, for all different ϕ, η is approximately zero when R is small; it suddenly increases when R is larger than the critical point R_c. It is clearly found that the capacity of the system is not the same for different ϕ.

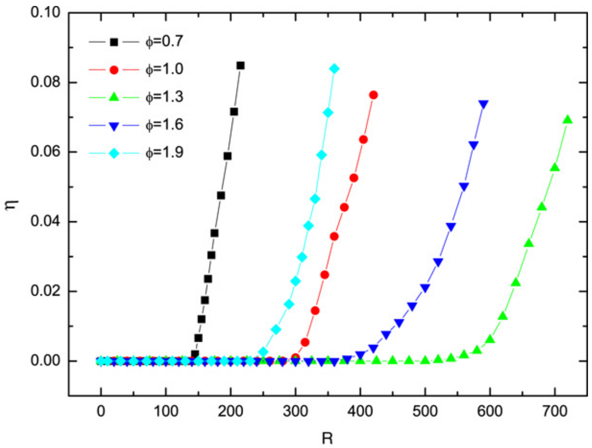

Fig. 1. The order parameter η versus R for different ϕ under the shortest path routing strategy. Average packet-delivering capacity of the network is $\langle C\rangle = 3$ and the network parameters are $N = 1000$, $m_0 = m = 3$.

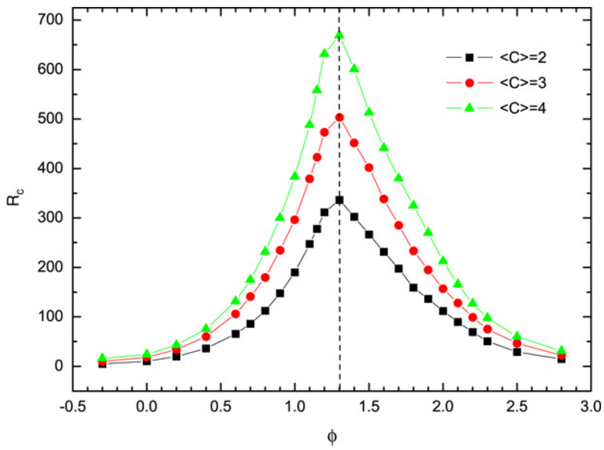

Fig. 2. The critical R_c versus ϕ for different $\langle C\rangle$ under the shortest path routing strategy. The network parameters are $N = 1000$, $m_0 = m = 3$.

Fig. 2 shows R_c versus ϕ for different $\langle C\rangle$ under the shortest path routing strategy. Interestingly, we find R_c is not a monotonic function of ϕ. There exists an optimal value of ϕ (positive) corresponding to the largest R_c, which means neither the uniform allocation nor the extremely uneven distribution can maximize network capacity. The emergence of optimal ϕ can be explained by betweenness centrality (BC) distributions in the scale-free network [33–35]. The BC of a node υ is defined as:

$$g(\upsilon) = \sum_{s\neq t} \frac{\sigma_{st}(\upsilon)}{\sigma_{st}}, \tag{5}$$

where σ_{st} is the number of shortest paths going from s to t and $\sigma_{st}(\upsilon)$ is the number of shortest paths going from s to t and passing through υ. BC gives an estimate of the traffic load on nodes when packets are forwarded following their shortest paths. For scale-free networks it has been shown that the relationship between betweenness centrality and degree obeys a power-law form: $g(k) \sim k^{\mu}$, and large-degree nodes endure a much heavier traffic load than that of small-degree nodes. Fig. 3 shows that the exponent $\mu = 1.33$ when the network parameters are $m_0 = m = 3$, $N = 1000$. Interestingly, we find this value of exponent is approximately equal to the optimal $\phi_{\text{opt}} = 1.3$ observed in Fig. 2.

To understand why $\phi_{\text{opt}} = \mu$ results in the maximal network capacity under the shortest path routing strategy, we investigate the queue length of a node $n(k)$ as a function of its degree k in the congested state ($R > R_c$). The queue length of a node is defined as the number of packets in the queue of that node. For ϕ small, i.e., $\phi = 0$, large-degree nodes do not get enough packet-delivering capacity while small-degree nodes have a redundant packet-delivering capacity which exceeds their actual load. As shown in Fig. 4(a), the queue length of large-degree nodes becomes longer and longer while at the same time small-degree nodes almost have no packets in their queue. Contrarily, if ϕ is very large, i.e., $\phi = 2.5$, most of the packet-delivering capacity is allocated to a few large-degree nodes and many small-degree nodes have too little packet-delivering capacity to fully dispose the load on them. As a result, packets continuously pile up on small-degree nodes (see Fig. 4(b)).

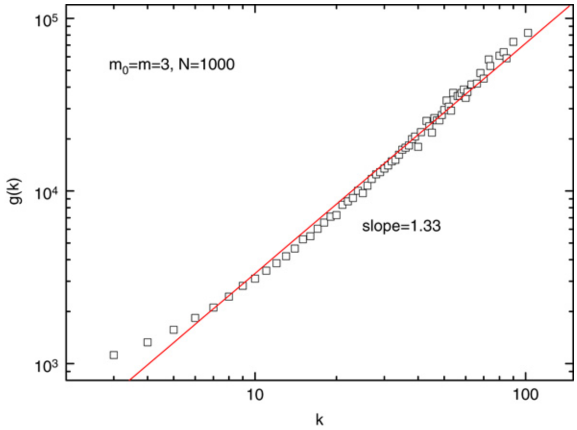

Fig. 3. Log–Log plot of betweenness centrality $g(k)$ versus degree k. The network parameters are $N = 1000$, $m_0 = m = 3$. The fitted line has a slope $\mu = 1.33$.

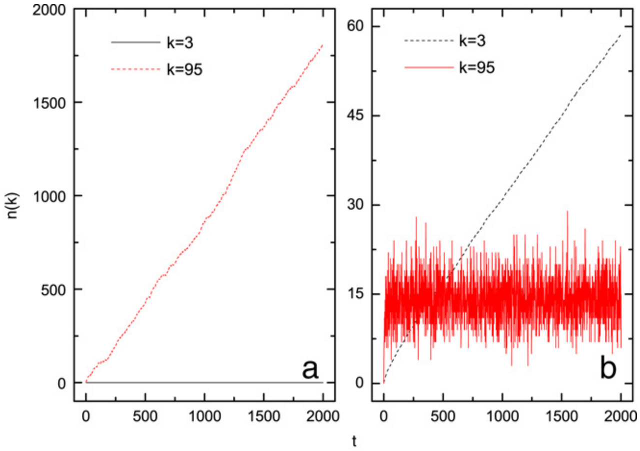

Fig. 4. Evolution of queue length $n(k)$ for different degree k under the shortest path routing strategy. $N = 1000$, $m_0 = m = 3$, $\langle C \rangle = 3$. The smallest degree $k = 3$ and the largest degree $k = 95$ in the BA network. (a) $R = 25 > R_c = 18$ for $\phi = 0.0$ and (b) $R = 100 > R_c = 47$ for $\phi = 2.5$.

In order to make full use of limited packet-delivering capacity and avoid congestion on a few nodes, the load distribution should be consistent with the packet-delivering capacity distribution, that is, $\phi_{\text{opt}} = \mu$. This average effect results in the maximal network capacity.

Next we investigate the behavior of R_c versus ϕ for different $\langle C \rangle$ under the local routing strategy. As shown in Fig. 5, there also exists an optimal value of ϕ corresponding to the largest R_c. For $\alpha = 0$, $\phi_{\text{opt}} = 1$. The optimal value of ϕ corresponding to different α is shown in Fig. 6(a). It is found that $\phi_{\text{opt}} \approx 1 + \alpha$ for the local routing strategy. According to the analysis in Ref. [17], the relationship between the queue length and degree is a power-law form: $n(k) \sim k^{1+\alpha}$ in the free flow state. To maximize the network capacity, the relationship between the packet-delivering capacity and degree also obeys the same power-law form. Furthermore, we study the maximum R_c as a function of α (Fig. 6(b)). One can find that there also exists nonmonotonous behavior with a peak at about $a = 0.2$.

4. Conclusion

In conclusion, we have investigated how to rationally allocate packet-delivering capacity onto nodes in the BA scale-free network when the sum of all the nodes' packet-delivering capacity is fixed. A tunable parameter is introduced, governing a node's packet-delivering capacity based on its degree. Interestingly, we find there exists an optimal value of parameter ϕ leading to the maximal network capacity. We provide some explanations for the emergence of optimal ϕ by investigating betweenness centrality distribution in the shortest path routing strategy and the queue length distribution in the local routing strategy. Our work may be helpful for designing a realistic communications network.

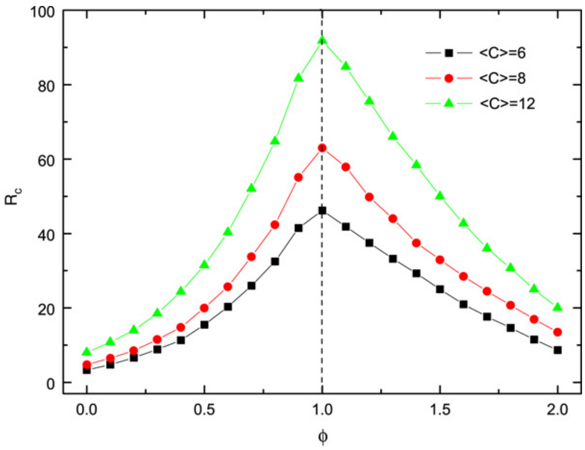

Fig. 5. The critical R_c versus ϕ for different $\langle C \rangle$ under the local routing strategy ($\alpha = 0$). The network parameters are $N = 1000$, $m_0 = m = 4$.

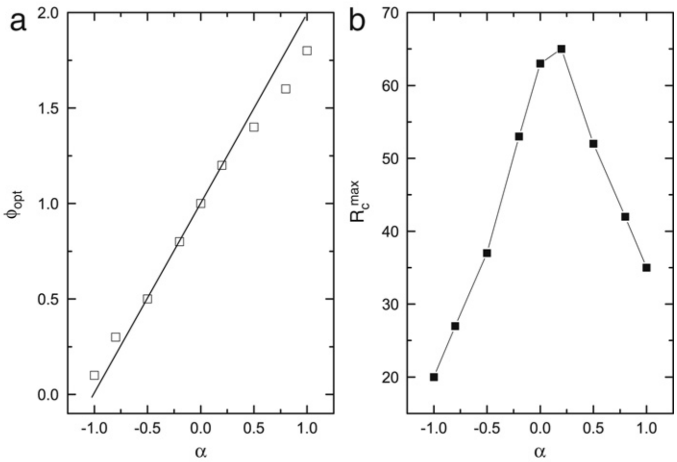

Fig. 6. (a) The optimal value of ϕ for different α under the local routing strategy. The line is the theoretical prediction. (b) The maximum R_c as a function of α. The network parameters are $N = 1000$, $m_0 = m = 4$. The average packet-delivering capacity of the network $\langle C \rangle = 8$.

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