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Age-based model for weighted network with general assortative mixing

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ABSTRACT

In this paper, we propose an evolutionary model for weighted networks by introducing an age-based mutual selection mechanism. Our model generates power-law distributions of degree, weight, and strength, which are confirmed by analytical predictions and are consistent with real observations. The investigation of the relationship between clustering and the connectivity of nodes suggests hierarchical organization in the weighted networks. Furthermore, both assortative and disassortative properties can be naturally obtained by tuning a parameter α, which controls the strength of age-based preferential attachments. Since the age information of nodes is easier to acquire than the degree and strength of nodes, and almost all empirically observed structural and weighted properties can be reproduced by the simple evolutionary regulation, our model may reveal some underlying mechanisms that are key for the evolution of weighted complex networks.

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1. Introduction

Many systems can be naturally described by complex networks with vertices representing individuals and edges representing interactions among individuals. Exploring evolutionary dynamics of complex networks have attracted much attention of scientific communities [1–4]. Much empirical evidence has demonstrated that most real networks share small-world and scale-free structural properties. Modeling and reproducing these common structural and dynamical properties have been deemed as a significant task for understanding the evolutionary dynamics of complex networks [5–17]. However, most real networks are weighted networks and far beyond the Boolean representations which would miss some important physical characters on edges. For instance, traffic amounts along edges and passing through vertices of communication and transportation systems are fundamental for a full description of these networks. In world-wide airport networks (WAN), each given edge weight (traffic) is the number of available seats on direct flight connections between the airports i and j. In the scientific collaboration networks (SCN), the nodes are identified by authors and the weight denotes the number of coauthored papers.

A weighted network is often described by a weighted adjacency matrix w_{ij}, which represents the weight on the edge connecting vertices i and j, with $i, j = 1, \ldots, N$, where N is the size of the network. The weights are symmetric ($w_{ij} = w_{ji}$) for undirected networks, which we will focus on. A natural generalization of connectivity in the case of weighted networks is the vertex strength defined as $s_i = \sum_{j\in\nu(i)} w_{ij}$, where the sum runs over the set $\nu(i)$ (neighbors of node i). This quantity is a natural measure of the importance or centrality of a vertex in the network. Most recently, the access to more complete

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empirical data and higher computation capability allow scientists to study the variation of the connection weights of many real networks [7,18–23]. As confirmed by measurements, weighted complex networks not only exhibit the scale-free degree distribution $p(k) \sim k^{-\gamma}$ with $2 \leq \gamma \leq 3$, but also the power-law weight distribution $p(w) \sim w^{-\theta}$ and strength distribution $p(s) \sim s^{-\eta}$ [19,21,22]. The strength highly correlated with the degree also displays scale-free property $s \sim k^{\beta}$ with $\beta \geq 1$ [19,23].

A number of evolutionary mechanisms have been presented to model real weighted networks. Barrat, Barthélemy, and Vespignani (BBV) presented an original model to study the growth of weighted networks [24,25]. The BBV model, based on the mechanisms of strength preferential attachment and weight dynamical evolution, can produce scale-free properties of degree, weight, and strength, but its assortative property (i.e., the hubs are primarily connected to less connected nodes), as observed in real technological and biological networks, differs from social networks like the SCN where the hubs are very likely to be linked together (i.e., assortative mixing). Recently, Wang et al. have studied the creation and reinforcement of internal connections in the weighted network evolution [26–29]. However, all these models incorporate the preferential attachment of the strength for reproducing scale-free properties. That is, a new added node is connected to a preexisting one with a probability exactly proportional to the strength of the target node. In reality, however, this absolute quantity information of agents is often unknown. In this perspective, Fortunato et al. recently introduced a criterion of network growth that explicitly relies on the ranking of nodes according to the prestige measure [30]. This rank-based model can mimic well the reality in many real cases that the relative information of agents is easier to acquire than their absolute information [31].

In this paper, we propose a weighted evolutionary model with an age-based mutual selection mechanism. It can mimic the reinforcement of internal connections and the evolution of many infrastructure networks with less information of vertices compared to previously proposed models. It is demonstrated that the generated networks recover scale-free distributions of degree, strength, and weight. Interestingly, this network evolution mechanism can also produce a hierarchical structure, and both assortative and disassortative properties, all of which have been empirically observed.

The paper is organized as follows: in Section 2 we describe the model in detail and analyze it mathematically. Simulation results of various scale-free properties are given in Section 3. The hierarchical structure and assortative mixing pattern are discussed in Section 4. Finally, the paper is concluded.

2. The model and analysis

The model starts from an initial $N_0 = m$ isolated node, each with initial age coefficient $h = 1$. At each time step t, a new isolated node, with age coefficient $h = t$, is introduced into the system. Then every existing node i preferentially selects m other nodes with the probability

$$\prod_{i \to j} = \frac{h_j^{-\alpha}}{\sum_k h_k^{-\alpha} - h_i^{-\alpha}}, \tag{1}$$

where h_i is the age coefficient of node i, and it's an age-based rank series. The parameter α which controls the strength of preferential attachment is a real number larger than 0. Note that the larger the age coefficient of the node is (the node is younger), the more difficult for it to gain new links. Considering the normalization requirement and that vertices are not permitted to connect themselves, the denominator of Eq. (1) is $\sum_k h_k^{-\alpha} - h_i^{-\alpha}$. Such selection is totally free and does not guarantee the creation of new links or an increase of edge weights between node pairs. Unless two nodes mutually select each other (in other words, unless they attract each other), there will be no change to the pair of nodes or their connection. If they do, then the weight of their link w is supposed to increase by 1. The degree k_i of any node i is still defined as the number of linked neighbors of i. Repeated interactions (links) only increase the edge weight. As a remark, w can be regarded as 0 if the nodes were not connected before, and the mechanism is globally implemented for all the nodes. After updating all the edge weights and node strengths, the growth process is iterated by introducing a new node, until the desired size of the network is reached. The mechanism of our model can well mimic the reality in many real cases. Take the SCN for example: scientists are more likely to collaborate with other experienced scientists, i.e., the scientists have long research age, and collaboration among scientists requires their common interest and mutual acknowledgement.

The model time is measured with respect to the number of nodes added to the network, i.e., $t = N - m$, and the natural time scale of the model dynamics is the network size N. We consider time t as a continuous variable using the continuous approximation. The parameters w, s, k are the functions of t and w_{ij} is updated only if node i and j select each other, and the dynamics function of weight evolution can be expressed as follows:

$$\frac{\mathrm{d}w_{ij}}{\mathrm{d}t} = m\frac{h_j^{-\alpha}}{\sum_k h_k^{-\alpha} - h_i^{-\alpha}} \times m\frac{h_i^{-\alpha}}{\sum_k h_k^{-\alpha} - h_j^{-\alpha}}. \tag{2}$$

The increment of strength s_i is contributed by the creation or reinforcement of internal connections incident with node i, which can be written as

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = \sum_j \frac{\mathrm{d}w_{ij}}{\mathrm{d}t}$$

$$= \sum_j \left(m^2 \frac{h_j^{-\alpha}}{\sum_k h_k^{-\alpha} - h_i^{-\alpha}} \times \frac{h_i^{-\alpha}}{\sum_k h_k^{-\alpha} - h_j^{-\alpha}} \right)$$
$$\approx \frac{m^2 h_i^{-\alpha}}{\sum_k h_k^{-\alpha}}. \quad (3)$$

We only consider the cases $\alpha < 1$, because when $\alpha < 1$, $\sum_k h_k^{-\alpha}$ is not a convergent series and the increment of strength tends to zero as t increases, which means that the evolution of the network converges and the network can reach a stable topological and weighted statistical property. Using the continuous approximation, Eq. (3) can be approximately solved by integrals. Therefore, the node strength evolution equation is

$$s_i(t) = \frac{m^2(1-\alpha)}{\alpha} \left(\frac{t}{i}\right)^{\alpha} - \frac{m^2(1-\alpha)}{\alpha}. \quad (4)$$

The knowledge of the time evolution of the strength quantities allows us to compute its statistical properties. Because i of $s_i(t)$ is selected randomly, i obeys homogeneously distribution in total $t+m$ nodes, i.e., $p(i) = 1/(t+m)$. The strength probability distribution can be written as

$$p(s,t) = \frac{\mathrm{d}p(s_i(t) < s)}{\mathrm{d}s} = \frac{t(s\alpha + m^2(1-\alpha))^{-1-1/\alpha}}{(t+m)(m^2(1-\alpha))^{-1/\alpha}}. \quad (5)$$

In the infinite size limit $t \to \infty$, the distribution is

$$p(s)_{t\to\infty} = \frac{(s\alpha + m^2(1-\alpha))^{-1-1/\alpha}}{(m^2(1-\alpha))^{-1/\alpha}}. \quad (6)$$

It was obvious that, when $s \gg m^2(1-\alpha)/\alpha$, $p(s)$ has a power-law distribution $p(s) \sim s^{-\eta}$, with $\eta = 1 + 1/\alpha$. Similarly, it can also obtain analytical expression for the statistical probability distribution of weights:

$$p(w,t) = \frac{t^{-(2\alpha-1)/\alpha}}{a(t+m)^2} \left(\frac{2\alpha - 1}{m^2(1-\alpha)^{-1/\alpha}}\right)^{-1/\alpha} w^{-1-1/\alpha}. \quad (7)$$

It is clear that $p(w) \sim w^{-\theta}$ with $\theta = 1 + 1/\alpha$.

3. Simulation result

In order to check the analytical predictions, numerical simulations for different values of α and m were performed. Fig. 1 shows the strength distributions for different parameter values. Numerical simulations are well consistent with analytical results. One can find that the power-law distribution exponent η of the strength is only related with α. But the distribution displays exponent corrections in the zone of low strengths, which are interestingly very similar with the empirical finding in some social networks [3]. The size of the exponential correction in the low strength zone is related with $m^2(1-\alpha)/\alpha$, which is clear in Fig. 1 and Eq. (6). Fig. 2 shows the weight probability distributions. It exhibits obviously scale-free behavior and the exponent agrees well with the theoretical predictions.

As confirmed by empirical measurements, weighted networks not only exhibit scale-free distributions of strength and weight, but also the power-law degree distribution $p(k) \sim k^{-\gamma}$ with $2 \le \gamma \le 3$. The strength highly correlated with the degree usually displays scale-free property $s \sim k^{\beta}$ with $\beta \ge 1$. To compare the model with the real network, the diversity of scale-free characteristics is simulated. The average strength s_i of vertices with degree k_i is shown in Fig. 3, which shows the scale-free property of $s \sim k^{\beta}$ as confirmed by empirical measurement. The exponent β here is around 1 and independent of the values of α and m when $\alpha < 1$. Thus, by using the correlation of $s \sim k$, we have

$$p(k) = p(s)\frac{\mathrm{d}s}{\mathrm{d}k} \sim p(s), \quad (8)$$

which indicates that the distribution of node degree is consistent with the distribution of node strength which has been shown in Fig. 1.

4. Clustering and correlation

The architecture of complex networks, imposed by the structure and administrative organization of the system, can not be exclusively characterized by the scale-free distribution properties. In fact, it is mathematically encoded in the various correlation existing among the properties of different vertices. For this reason, a set of topological quantities have been introduced to character the network architecture. The widely used quantities are the clustering coefficient and the

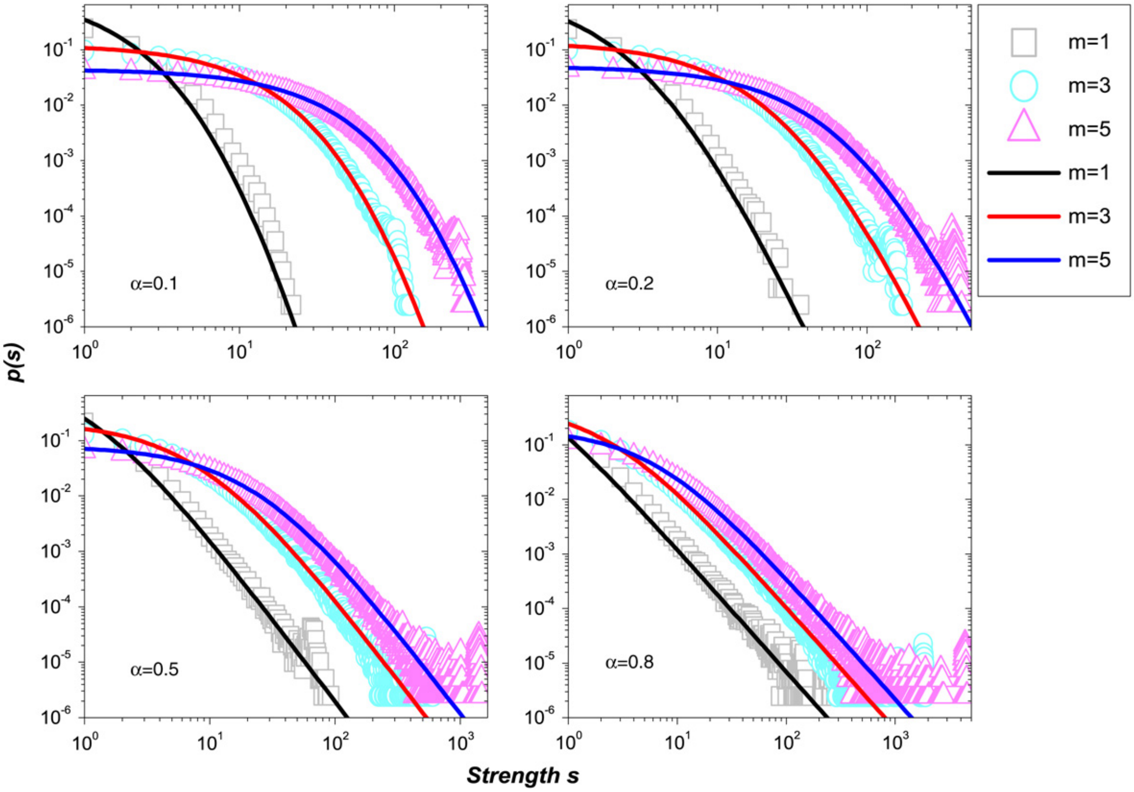

Fig. 1. (Color online) Strength probability distributions of both computer simulation and analytical result of our model with different parameters $\alpha = 0.1$, $\alpha = 0.2$, $\alpha = 0.5$, and $\alpha = 0.8$. The solid lines represent analytical result. All the data are averaged over 100 independent runs of network size $N = 4000$.

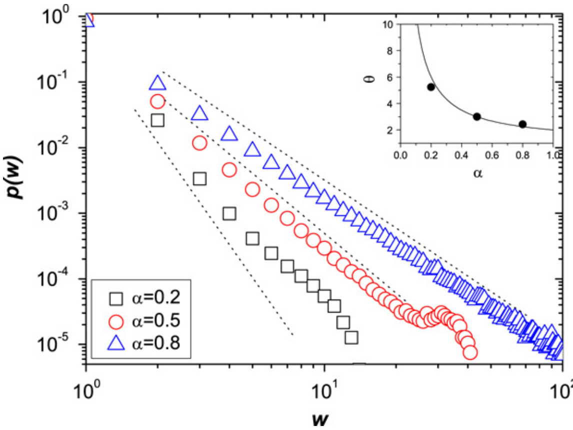

Fig. 2. (Color online) The weight probability distributions $p(w) \sim w^{-\theta}$ with different α. As shown in its inset, the data fitting also gives values of θ (full circles) as predicted by analytical calculation (line). All the data are averaged over 100 independent runs of network size $N = 4000$, and $m = 5$.

assortative mixing pattern (or the degree–degree correlation of nodes). To better understand the architecture structure, we studied the clustering of vertices and the degree–degree correlation by choosing different values of α and m.

Using the method proposed by Watts and Strogatz, we define c_i as the clustering of vertex i, which measures the local cohesiveness of the network in the neighborhood of the node. The average of all nodes gives the network clustering coefficient C, which describes the statistics of the density of connected triples [3,32]. Further information can be gathered by inspecting the average clustering coefficient $C(k)$ restricted to classes of vertices with degree k. For the degree–degree correlation, we use the average nearest-neighbor degree $k_{nn,i}$ to measure the degree–degree correlation of vertex i and its neighbors. $k_{nn}(k)$ is defined as the average over classes of vertices with degree k. When correlations are present, two main classes of possible correlations can be identified: assortative behavior if $k_{nn}(k)$ increases with k, which indicates that large degree vertices are preferentially connected with other large degree vertices, and disassortative behavior if $k_{nn}(k)$ decreases

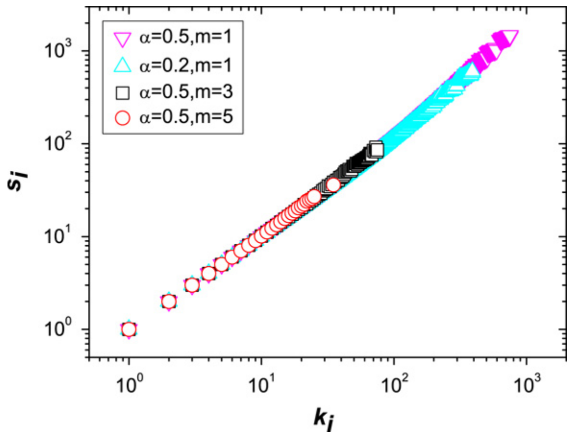

Fig. 3. (Color online) The average strength s_i of vertices with degree k_i (log–log scale). The exponent of the distribution doesn't change with the parameters m and α. All the data are averaged over 100 independent runs of network size $N = 4000$.

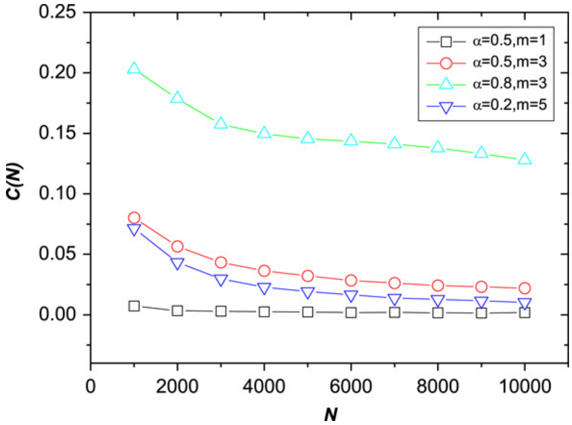

Fig. 4. The clustering coefficient C versus N with different parameters.

with k, which denotes that links are more easily built between large degree vertices and small ones. A simpler measure to quantify this structural property is the assortative mixing coefficient proposed by Newman [33]:

$$r = \frac{M^{-1}\sum_{i=1}^{M} j_i k_i - \left[M^{-1}\sum_{i=1}^{M}\frac{1}{2}(j_i + k_i)\right]^2}{M^{-1}\sum_{i=1}^{M}\frac{1}{2}(j_i^2 + k_i^2) - \left[M^{-1}\sum_{i=1}^{M}\frac{1}{2}(j_i + k_i)\right]^2}, \tag{9}$$

where j_i, k_i are the degrees of vertices at the ends of the ith edges, with $i = 1, \ldots, M$ (M is the total number of edges in the observed graph). The values of this quantity are restricted in the interval $[-1, 1]$, and moreover, positive values denote assortative and negative values denote disassortative.

Fig. 4 shows the clustering coefficient $C(N)$ as a function of the network size N for different values of α and m. $C(N)$ converges soon as the network size increases. A number of real networks possess a hierarchical structure property, which is quantified by the dependence of the average clustering $C(k)$ of nodes with the same degree on the degree k [34]. Empirical results demonstrate that $C(k)$ is a decreasing function of k with a fat tail in the large-degree range. This behavior indicates that low-degree nodes generically belong to well connected clusters while the neighbors of high-degree nodes belong to many different communities which are not directly connected, namely the hierarchical structure. Fig. 5 shows $C(k)$ depending on k under the different values of α. One can find that when α is close to 1, $C(k)$ versus k displays a flat head with a fat tail in the log–log plot, which is similar to the real observations and exhibits the hierarchical structure [34]. On the other hand, as α decreases from 1, $C(k)$ becomes a non-monotonic function of k, which indicates that the age-strength parameter α has strong influences on the formation of the hierarchical organization of the weighted network.

The correlation of $k_{nn}(k)$ and k for different values of α is shown in Fig. 6. The increasing trend and decreasing trend of $k_{nn}(k)$ depending on k both exist, which implies the general patterns of both assortative and disassortative mixing generated by our model. To measure the assortative mixing pattern more simply and intuitively, we study the coefficient r. As shown

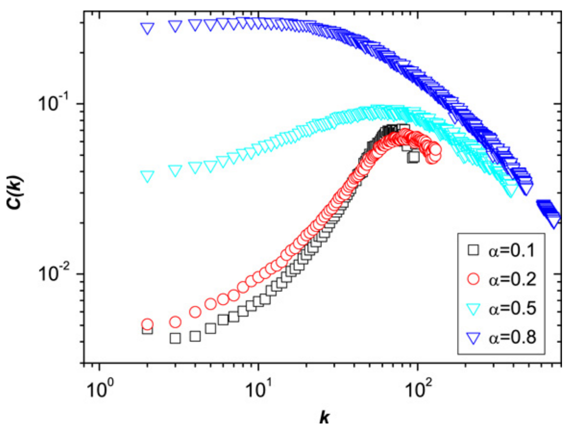

Fig. 5. (Color online) The distribution of $C(k)$ as a function of k with different α (log–log scale). $m = 3$ is focused on. All the data are averaged over 100 independent runs of network size $N = 4000$.

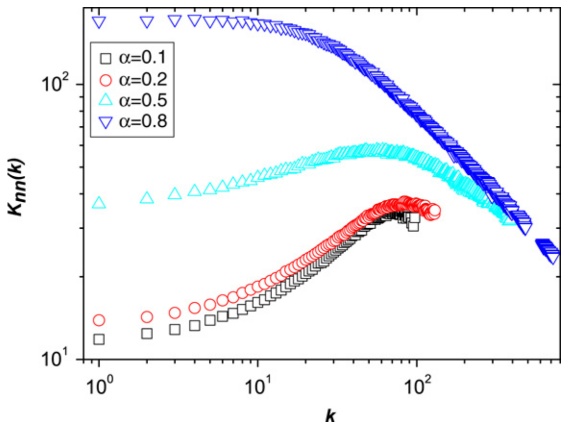

Fig. 6. (Color online) The distribution of $k_{nn}(k)$ as a function of k with different α. $m = 3$ is focused on. All the data are averaged over 100 independent runs of network size $N = 4000$.

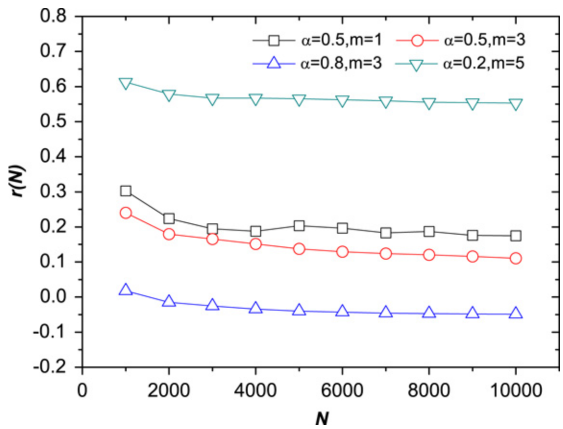

Fig. 7. The mixing coefficient r versus N with different parameters.

in Fig. 7, $r(N)$ converges soon as the network size N increases for different values of α and m. The dependence of r on both α and m is shown in Fig. 8. It can be found that the mixing coefficient r only changes with α, and nearly independent of m. With the increase of α, r can change from positive values to negative values, which is consistent with the results of assortative mixing reflected by $k_{nn}(k)$.

Empirical studies have found that many social networks (e.g., SCN) present assortative behaviors while technological networks (e.g., Internet, WAN) present disassortative behaviors. In general social networks, take SCN for example, when

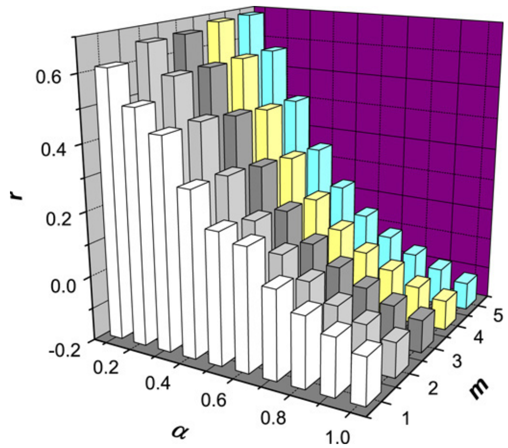

Fig. 8. The mixing coefficient r depending on both m and α with $N = 4000$.

scientists want to get collaboration with other scientists, the experience of the other scientists, i.e., the scientific age, plays important roles. Besides, there are some other factors (e.g., area distinction, social ability, history collaboration, etc) that may affect the cooperation of two scientists. Thus, there is an obvious need to model weighted networks by considering possible mechanisms other than the node-strength-based preferential attachment. Few previously proposed weighted models can generate both assortative and disassortative mixing properties. Compared to previous models, our model by simply considering the age-based mutual selection mechanism can characterize well the origin of both mixing patterns. The parameter α governs the strength of the age-based preferential attachment. For smaller α, younger nodes have higher chances to select each other for possible interaction, which will lead to denser connectivity among younger nodes and thus favor the emergence of an assortative mixing pattern. On the other hand, larger α promotes the formation of connections between younger nodes and older nodes. Moreover, old nodes usually have more neighbors and younger nodes in reverse have fewer neighbors. Hence, the fact that more connections are built between the two groups with different degrees results in the emergence of a disassortative pattern. From the above analysis, the intensity of the age-based preferential attachment plays the key role in the mixing difference of our weighted network.

5. Conclusion

To summarize, in this paper we propose an age-based mutual selection model for weighted evolutionary networks. The model networks recover scale-free distributions of degree, strength, and weight. Furthermore, our model generates the nontrivial clustering and assortative mixing patterns. By changing the values of α, both assortative and disassortative properties can be simply reproduced, which implies the significance of our proposed age-based selection mechanism, since the origin of the assortative mixing difference between social networks and other kinds of networks is still a challenging question so far. More significantly, the age-based selection mechanism is very natural and the age information is easily known by individuals in a complex network. From this viewpoint, our model may make sense in exploring simple and unified underlying mechanisms existing in various weighted complex networks.

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