

# Self-questioning games and ping-pong effect in the BA network

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## Abstract

In this paper, we study the evolutionary Prisoner's Dilemma Game (PDG) and the Snowdrift Game (SG) with a self-questioning updating mechanism in the Barabási–Albert (BA) network. Although this self-questioning mechanism does not show much advantages in sustaining the cooperative behavior comparing to the existing models, it can produce interesting non-monotonic phenomena in numerical simulations. Furthermore, this new model has avoided the system from being enmeshed in a globally defective trap, which is a shortcoming of the existing models based on the learning mechanisms. It is found that in certain cases, the so called “Cooperative Ping-pong Effect” occurs in both the two games and plays an important role in the behaviors of the whole system. This new model shows non-trivial characters comparing to the previous work and is worthy of further studies.

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## 1. Introduction

Cooperation is ubiquitous in economic and social systems [1]. These systems are filled with selfish individuals who try to maximize their own benefits. But being contrary to the view of the Darwinian selection, cooperation becomes the main behavior of these systems. The emergence of cooperation in selfish circumstance is to be understood and has attracted much attention from physicists recently. Game theory, together with its extensions, provides a useful framework to investigate this problem [2–5]. The Prisoner's Dilemma Game (PDG) [6] and the Snowdrift Game (SG) [7], as general metaphors, are often used in this field. In the original games of PDG and SG, each of the two players may choose either to cooperate (*C*) or defect (*D*) when they encounter. If they both choose *C* (or *D*), each will get a payoff of *R* (or *P*). If one of them chooses *C* while the other chooses *D*, the defector will get a maximum payoff of *T*, and the cooperator gets *S*. The payoff matrix can be described as

$$\begin{array}{cc|cc} & & \mathbf{C} & \mathbf{D} \\ \mathbf{C} & & R & S \\ \mathbf{D} & & T & P. \end{array} \quad (1)$$

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Game theory has restricted a precondition of  $2R > T + S$ , for mutual cooperation will benefit the society most, thus corresponds to the largest total payoff. In the PDG, the rank of the four payoff values is  $T > R > P > S$ , while in the SG it is  $T > R > S > P$ . So the SG is more favorable to sustain cooperative behavior.

But the original PDG and SG can only sustain an unstable cooperative behavior, which is opposite to the observations in the real world. Extensions based on the original games are proposed to match the high cooperation frequency observed in real world systems. Such as the “tit-for-tat” strategy, can remarkably promote the cooperative behavior [4,8]. Recently, physicists are trying to relate the evolutionary cooperative behavior to the spatial structure [9–14]. As the studies on the complex networks are greatly developed, evolutionary games are naturally considered on the networks [13–16]. Usually three types of networks are concerned in this field, i.e., the “Regular Network”, the “Small-World Network” and the “Scale-Free Network”.

In the evolutionary games with network structure, each player occupies a node of the network and plays the game only with his immediate neighbors. In each round, the obtained total payoff of a player is the sum of the payoffs obtained in all the encounters with his immediate neighbors. In the next round, the players will simultaneously update their strategies ( $C$  or  $D$ ) according to certain rules. The common-used updating rules include the deterministic rule proposed by Nowak and May [9], and the stochastic evolutionary rule by Szabó and Töke [11] etc. In this paper, we adopt the idea of the latter one and focus our work on the games in scale-free networks.

In the previous work of Szabó and Töke, players update their strategies by learning from their neighbors. Each player will randomly choose one out of the staff of his immediate neighbors and then adopt the selected neighbor’s strategy ( $C$  or  $D$ ) with a certain probability  $W$ . The probability  $W$  is usually determined by the total payoffs of the two players. As the players aim at maximizing their own benefits, therefore, if the selected neighbor has a higher payoff, it is quite likely for his strategy to be adopted, and vice versa. For example, player  $i$  is going to learn from player  $j$ . Suppose that  $M_i$  and  $M_j$  are the total payoffs of player  $i$  and  $j$ . The probability  $W$  given by Szabó and Töke is

$$W_{i \rightarrow j} = \frac{1}{1 + \exp[(M_i - M_j)/K]}. \quad (2)$$

Here the parameter  $K$  characterizes the noise effects and is introduced to permit irrational choices. Eq. (2) has a manner of statistical physics and is generally adopted in previous works.

Most models of the evolutionary games adopts the learning mechanism as mentioned above. Players update their strategies by learning from their neighbors. Recently, Wang etc. have proposed a memory-based SG on networks [17] which abandons the learning mechanism. Instead, a self-questioning mechanism and a memory-based updating rule are presented in Ref. [17]. As a extension of this work, we study on the evolutionary PDG and SG with self-questioning mechanism and stochastic evolutionary rule, mainly on the scale-free network. Interesting results are obtained and shown in the following parts of this paper.

In the following section, we describe our model in details. In Section 3, simulation results and statistical analysis are provided. In Section 4, we will give further discussion on the PDG. In Section 5, the work is concluded.

## 2. The model

We firstly establish a scale-free network by using the BA model. The BA network is the most simple and general network with scale-free characters. Starting from  $m_0$  nodes,  $m$  new nodes are added to the network at each time step. The newly added nodes connect to the old ones according to a preferential attachment mechanism in such a way that the probability  $\Pi_i$  of connecting to a existing node  $i$  is

$$\Pi_i = \frac{k_i}{\sum_j k_j},$$

where  $k_i$  is the degree of node  $i$ , i.e., the number of its neighbors; the sum runs over all the existing nodes in the current network.

We consider the evolutionary PDG and SG on the BA network. Without losing generality, we adopt the simplified games with a single payoff parameter following previous works [9–11]. The payoff matrix is

$$\begin{array}{cc|cc}
 & \text{PDG} & & \text{SG} \\
 & \text{C} & \text{D} & \text{C} & \text{D} \\
 \text{C} & 1 & 0 & 1 & 1-r \\
 \text{D} & b & 0 & 1+r & 0,
 \end{array} \tag{3}$$

where  $b \in (1, 2)$  and  $r \in (0, 1)$ , are the only payoff parameters in the PDG and the SG, respectively. In this case, the rank  $T > R > S > P$  for the SG is strictly satisfied. As for the PDG, the rank is  $T > R > P = S$ . We will use this matrix to study on the typical characters of the PDG, as a statement under ideal conditions. In Section 4, we will discuss on the situation when  $P$  is positive but significantly below unity.

In the evolution process, we consider simultaneously the self-questioning mechanism and the stochastic evolutionary rule. In each time step, players get payoffs through the game on the basis of the payoff matrix. Then each player calculates a virtual payoff by self-questioning, i.e., to adopt its anti-strategy and play a virtual game with its neighbors who keep their strategies unchanged, then getting a virtual payoff [17]. By comparing the real payoff and the virtual payoff, players will find out whether their current strategies are advantageous. In the next round, player  $i$  will change its current strategy to its anti-strategy with probability  $W_i$ , in which

$$W_i = \frac{1}{1 + \exp[(M_i - M'_i)/K]} \tag{4}$$

The  $M_i$  and  $M'_i$  above are the real and virtual payoff of player  $i$ , respectively. Similar to Eq. (2), the parameter  $K$  characterizes the noise effects.

### 3. Simulation results and statistical analysis

The key quantity for characterizing the cooperative behavior of the system is the frequency of cooperation,  $f_c$ , defined as the proportion of cooperators among all the players. In previous works, a general conclusion for both the PDG and the SG is that  $f_c$  will descend as the payoff parameter increases, for larger payoff parameter gives more benefits to the defectors. The simulation results of our model in regular networks and small-world networks do not show much non-trivial results. In this paper, we focus on the evolutionary games in the scale-free networks, constructed by the BA model. Simulation results for both the PDG and SG are shown in the following figures. The games are played among 10 000 players, occupying the nodes of a BA network. The simulation results are obtained from the last 1000 time steps of total 10 000 time steps of evolution. Each data point results from an average over 20 individual runs. In the initial states, cooperators and defectors are uniformly distributed among all the players.

Fig. 1(a) and (b) show the frequency of cooperation  $f_c$  as a function of the payoff parameters, for the PDG and SG, respectively. We fix the noise parameter as  $K = 0.2$  and plot the curves under different values of  $m$ , which characterizes the averaged degree of the BA network. With our evolutionary mechanism, the PDG and SG show totally different features. Fig. 1(a) shows that when  $m$  is small,  $f_c$  is a monotonic descending function of  $b$ . But as  $m$  increases, non-monotonic features of  $f_c$  come to appear. When  $b$  increases from a small value,  $f_c$  descends with it firstly, and drops off to a minimum level. After that, as  $b$  increases,  $f_c$  ascends gradually and finally reaches a fixed value of  $f_c = 0.25$ . While in Fig. 1(b), curves under different values of  $m$  have similar features, being all 180°-rotational symmetrical about the point (0.5, 0.5).

To understand the features shown in Fig. 1, we carry out statistical analyses as following. Because the mean-field hypothesis is not always fit for the evolutionary games on networks, the following statistical analyses can only be qualitative. Considering the payoff matrix in Eq. (1), if a player has  $n_c$  cooperative neighbors and  $n_d$  defective neighbors, its total payoff is  $P_C = n_c R + n_d S$  for choosing C, or  $P_D = n_c T + n_d P$  for choosing D.

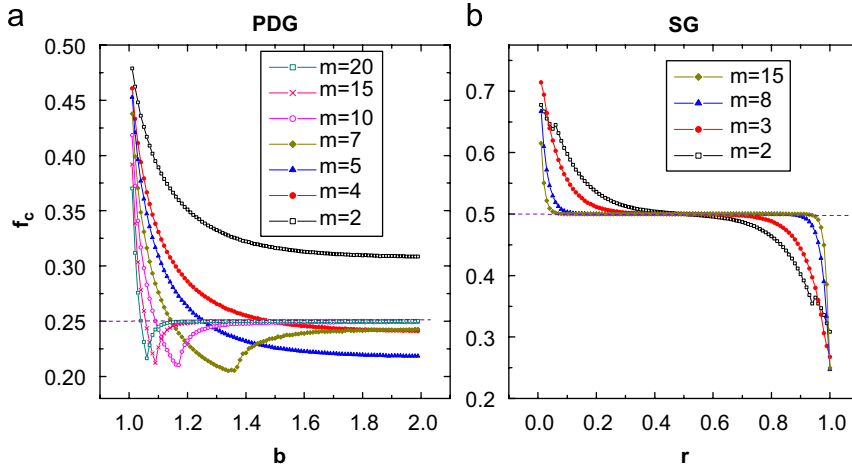


Fig. 1. (Color online) The frequency of cooperation  $f_c$  as a function of the payoff parameter, in the BA networks with different values of  $m$ : (a) for the PDG and (b) for the SG. The noise parameter is fixed as  $K = 0.2$ .

Then

$$P_C - P_D = n_c(R - T) + n_d(S - P) = \begin{cases} n_c(1 - b) & \text{for the PDG,} \\ n_d(1 - r) - n_cr & \text{for the SG.} \end{cases}$$

Imaging a localized block in the network, a player is surrounded by  $k$  neighbors among which the cooperators have a proportion of  $f$  while the defectors have the rest fraction  $1 - f$ . For the player,  $n_c = kf$ ,  $n_d = k(1 - f)$ , then

$$P_C - P_D = \begin{cases} kf(1 - b) & \text{for the PDG,} \\ k(1 - f - r) & \text{for the SG.} \end{cases} \tag{5}$$

At a time, supposing the player itself will choose  $C$  with a probability of  $\rho$ , or choose  $D$  with probability  $1 - \rho$ . In the next time step, the probability for the player to choose  $C$  should be

$$\rho_c = \rho \left( 1 - \frac{1}{1 + \exp[(P_C - P_D)/K]} \right) + (1 - \rho) \frac{1}{1 + \exp[(P_D - P_C)/K]} = \frac{1}{1 + \exp[-(P_C - P_D)/K]}$$

Considering Eq. (5), for the PDG,

$$\rho_c = \frac{1}{1 + \exp[-kf(1 - b)/K]} \tag{6}$$

for the SG,

$$\rho_c = \frac{1}{1 + \exp[-k(1 - f - r)/K]} \tag{7}$$

Eqs. (6) and (7) are both independent of  $\rho$ , which indicates that the probability for the player to cooperate in the next time step is totally decided by its surroundings, i.e., the ‘‘local cooperative frequency’’  $f$ . It is a noticeable result that although the players pay attentions to themselves, the strategies they adopt are totally decided by their neighbors. It indicates that the self-questioning is not simply an introverted action but

contains amplitude information about the circumstances. The updating mechanism in this paper is effective and feasible.

In Eq. (5), as  $b > 1$ ,  $P_C - P_D$  for the PDG is always negative. That is to say, in the PDG, defectors always have advantages to cooperators. Larger values of  $b$  and  $k$  will give more advantages to the defectors according to Eq. (5). Hence, in Fig. 1(a), we can find the main tendency for  $f_c$  is to descend with  $b$  or  $m$ .

But when the cooperative behavior is reduced to a certain extent, a special phenomenon comes to occur. When  $f_c$  is low enough, masses of defectors emerge in the system. That means, localized blocks filled with connective defectors are formed in the network. For example, at time  $t$ , the local cooperative frequency in such a block is  $f(t) = 0$ . In this case, Eq. (6) turns to

$$\begin{aligned}\rho_c(t+1) &= \frac{1}{1 + \exp[-kf(1-b)/K]} \\ &= \frac{1}{1 + e^0} \\ &= \frac{1}{2}.\end{aligned}$$

This happens to all the players in the defective mass. As a result, nearly half of the defectors turn to cooperators in the next time step, and the local cooperative frequency  $f(t+1) \approx 0.5$ . After that, at time  $t+2$ ,

$$\begin{aligned}\rho_c(t+2) &= \frac{1}{1 + \exp[-kf(1-b)/K]} \\ &\approx \frac{1}{1 + \exp[k(b-1)/2K]}.\end{aligned}$$

For a mass with high connectivity, i.e., large  $k$ ,

$$\rho_c(t+2) \rightarrow 0.$$

So at time  $t+2$ , all the players in the block defect again. It returns to the state at time  $t$ . As this recycling process repeats, the time series for the local cooperative frequency  $f_c$  is

$$0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, \dots$$

As shown in Fig. 2(b),  $f_c$  switches ceaselessly between the two values of 0 and 0.5. Being averaged over time, the observed value would be  $f_c = 0.25$ . This has explained the non-monotonic parts of the curves in Fig. 1(a). As the cooperative behavior being inhibited to a low level, such local effect comes to occur in the system, which has elevated the global frequency of cooperation. Until this local effect diffuses all over the network, the whole system begins to switch synchronously between the state of uniform defection and random choice. The global value of the cooperation frequency is fixed as  $f_c = 0.25$ , represented by the dashed horizontal line in Fig. 1(a).

This process mentioned above can be regarded as a “Cooperative Ping-pong Effect”, which has earlier been studied in the field of traffic flow [18–22]. Here “cooperative” means coherent. It describes a system switching back and forth between two different states. Within the system, elements are synchronized and switch between the states coherently. The ping-pong effect has more practical meanings in the evolutionary games than in the traffic flow. In fact, it reflects the fact that players are drifting with the tide. In order to obtain the maximized benefits, they change choices continually. In some cases, the whole system may be enmeshed in the vibration and cannot escape, and then the ping-pong effect occurs.

Generally speaking, the ping-pong effect, as shown in Fig. 2, would induce wastes of resources in the vibrating process. So in the traffic flow and other fields, it should be avoided. However, in the evolutionary games, it has an obvious advantages. As mentioned above, when the cooperative behavior is inhibited to quite a low extent, it could be partly retrieved by the ping-pong effect. Especially in a globally defective state, as all the neighbors are defectors, with the learning mechanism in previous works, the system could not escape from the defective trap at all. The whole system will be frozen in such a defective state. While in our model, with the self-questioning mechanism, although it results in a ping-pong vibration finally, the shortcoming of the previous works has been avoided and the cooperative behavior is sustained right along.

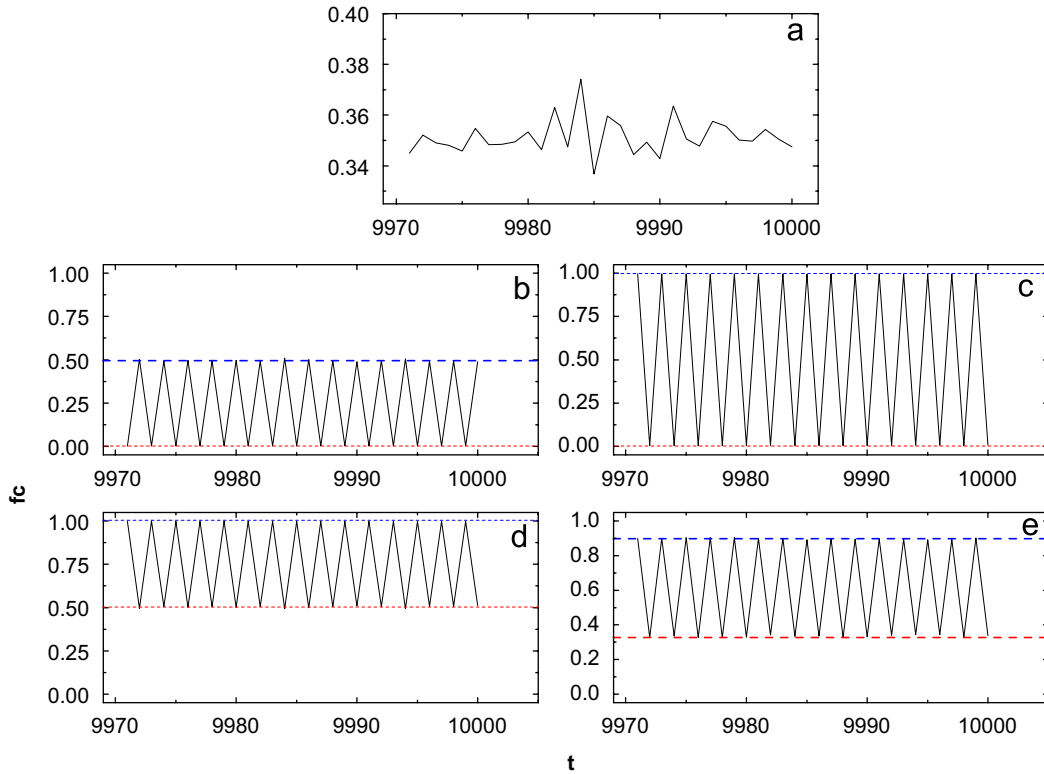


Fig. 2. (Color online) Time series for the frequency of cooperation, showing different types of the final states with different parameters: (a)  $m = 2, K = 0.2, b = 1.2$  for the PDG, in this case, no ping-pong effect occurs, (b)  $m = 10, K = 0.2, b = 1.5$  for the PDG, (c)  $m = 2, K = 0.2, r = 0.5$  for the SG, (d)  $m = 2, K = 0.2, r \rightarrow 0$  for SG, and (e)  $m = 6, K = 2.0, r = 0.2$  for the SG.

The discussion above is centered on the PDG. For the SG, the ping-pong effect is even more familiar. Previously in Ref [17], similar phenomenon has been observed in some special cases. In Eq. (5), for the SG,

$$P_C - P_D = k(1 - f - r). \tag{8}$$

Considering the symmetrical point  $r' = 1 - r$  for Eq. (8), then

$$\begin{aligned} P'_C - P'_D &= k(1 - f - r') \\ &= k[1 - f - (1 - r)] \\ &= k[1 - (1 - f) - r], \end{aligned}$$

i.e.,

$$P'_D - P'_C = k(1 - f_d - r), \tag{9}$$

where  $f_d = 1 - f$  is the frequency of defection. Comparing Eqs. (8) and (9), it is easy to find that when the transform  $r \rightarrow r' = 1 - r$  occurs, the situations of  $C$  and  $D$  interchange fitly, thus the frequency of cooperation  $f \rightarrow f' = 1 - f$ . As a result, all the curves in Fig. 1(b) pass through the point (0.5,0.5) and are all 180°-rotational symmetrical about it.

The ping-pong effect is familiar in the SG. It happens in the states represented by almost any of the data points in Fig. 1(b). For example, when  $r = 0.5$ , it seems that  $f = 0.5$  could be a steady solution of Eq. (7). But it is not always so simple. Similar analysis as above could be carried out on Eq. (7), and then give different final states with different parameters, as shown in Fig. 2. Fig. 2(c) exhibits the time series of  $f_c$  for the SG when ( $m = 2, K = 0.2, r = 0.5$ ), which shows an obvious coherent ping-pong effect vibrating between 0 and 1.

Fig. 2(d) shows the final state for ( $m = 2, K = 0.2, r \rightarrow 0$ ). In this case, Eq. (7) approaches to

$$\rho_c \rightarrow \frac{1}{1 + \exp[-k(1-f)/K]}$$

With a large value of  $k/K$ ,  $\rho_c \rightarrow 1$  when  $f < 1$ , the system switches to a globally cooperative state. Then in the next time step,  $f = 1, \rho_c \rightarrow 0.5$ , and so on.  $f_c$  vibrates between 0.5 and 1, as shown in Fig. 2(d). The averaged value of  $f_c$  is 0.75, which is the limit for all the curves in Fig. 1(b) on the left end.

The ping-pong effects discussed above are all globally coherent, i.e., the whole network is engaged into the vibration. In fact, it is possible for the ping-pong effect to occur only in local parts of the network. Fig. 2(e) shows the final state for  $m = 6, K = 2.0, r = 0.2$ . In this case, the ping-pong effect could not reach a globally coherent situation. It may occur only in local parts of the network, or occurs everywhere but could not be synchronized in a global scale, or happens between two states closer to each other. As a result, the vibration of  $f_c$  is relatively weakened. Furthermore, with extreme parameters, the ping-pong effect could be highly inhibited. For example, when  $r = 0.5$ , with an extremely large  $K$ , the ping-pong effect could be quite weak and  $f = 0.5$  became more like a steady solution of Eq. (7), similar to the feature of the final state shown in Fig. 2(a). But in this case, the adaptive evolution of the game will be inhibited by the large noise, too.

Now we discuss on the influences of the noise parameter  $K$ . As mentioned before,  $K$  characterizes the noise in the game and is introduced to permit irrational choices. Usually smaller  $K$  corresponds to stronger deterministic tendency in choosing strategies, and large  $K$  corresponds to random choices. When  $K \rightarrow 0$ , Eq. (4) approaches to

$$W_i = \begin{cases} 1, & M_i < M'_i, \\ 0, & M_i > M'_i. \end{cases}$$

Players will deterministically adopt the strategy related to the larger payoffs. On the other hand, when  $K \rightarrow +\infty$ ,  $W_i \rightarrow 0.5$ , which means the players will randomly make choices.

Fig. 3 shows the dependence of the cooperation frequency  $f_c$  on the noise parameter  $K$ , in the BA networks with different values of  $m$ . In each curve in Fig. 3(a) for the PDG, as  $K$  increases to a large value,  $f_c$  approaches to 0.5, as has been discussed above. When  $m$  is not too small, non-monotonic features are exhibited with small values of  $K$ . This non-monotonic phenomenon is induced by the ping-pong effect. As in the PDG, defectors always have advantages to cooperators, thus a deterministic-updating system will inhibit the cooperative behavior. When  $K$  is reduced to a certain value, cooperative behavior is inhibited seriously, and defective mass will emerge in the network, then the ping-pong effect comes to occur. As  $K$  decreases, the ping-pong effect develops from a local behavior to a globally coherent one. In this process,  $f_c$  retrieves continuously and finally reaches the value of 0.25 as analyzed before.

The emergence of the ping-pong effect in the SG is sensitive to the influence of  $K$ . Fig. 3(b) and (c) show the dependence of  $f_c$  on  $K$  for the SG when  $r = 0.2$  and  $r = 0.8$ , respectively. When  $K \rightarrow 0$ , the updating rule of the evolutionary game is deterministic. It induces a ping-pong effect between the globally cooperative and defective state. The averaged value for  $f_c$  is 0.5, which is the convergence point of the curves in Fig. 3(b) and (c) on the left end. As  $K$  increases, the ping-pong effect is inhibited gradually, that the low boundary of  $f_c$  is elevated and the high boundary descended. In Fig. 3(b), it is exhibited as an ascending section in each of the curves. After  $f_c$  reaches the peak, a sudden decline occurs immediately. Comparing the curves for different values of  $m$  in Fig. 3(b), we can find the sudden declines happen with similar values of  $m/K$ , where the ping-pong effect disappears all at once. When  $m/K$  reaches a critical value, according to Eq. (7), the system abandons to vibrate between two different states, but keeps close to a steady one. After that, the ping-pong effect is totally inhibited by large noises.

Fig. 4 shows the dependence of  $f_c$  on the parameter  $m$ , which represents the averaged degree of the network. Non-monotonic phenomenon is exhibited in Fig. 4 obviously. For the PDG, larger  $m$  benefits the defectors more, so  $f_c$  decreases with  $m$  in Fig. 4(a). When the payoff parameter  $b$  is not too small, large  $m$  induces the emergence of the ping-pong effect, which is reflected in Fig. 4(a) as the ascending parts of the curves. In Fig. 4(b), in order to show the details,  $m$  is not necessary to be an integer. For example, when establishing a BA network, to generate 2.7 edges for a new node can be implemented as generating 2 edges firstly and then



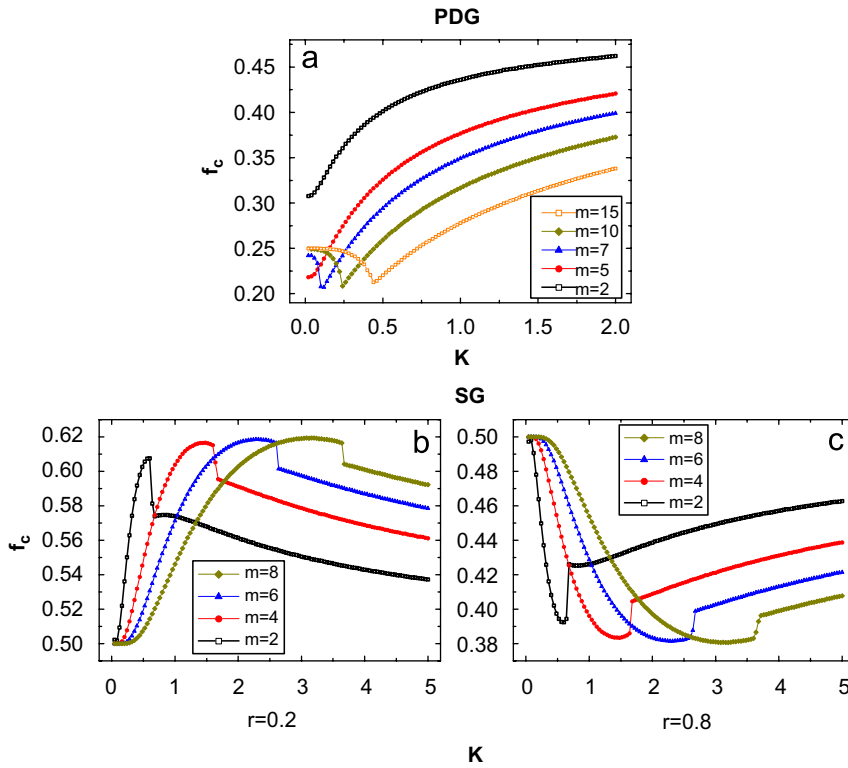


Fig. 3. (Color online) The frequency of cooperation  $f_c$  as a function of the noise parameter  $K$ , in the BA networks with different values of  $m$ : (a)  $b = 1.2$  for the PDG, (b)  $r = 0.2$  for the SG, and (c)  $r = 0.8$  for the SG.

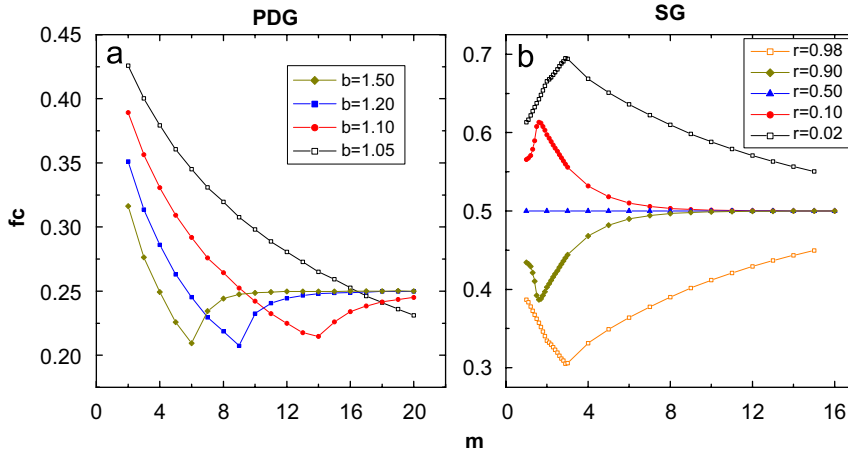


Fig. 4. (Color online) The frequency of cooperation  $f_c$  as a function of the parameter  $m$ , with the fixed noise parameter  $K = 0.2$ : (a) for the PDG, and (b) for the SG.

generating the third one with probability 0.7. In Fig. 4(b), for  $r < 0.5$ ,  $f_c$  increases firstly with the increasing connectivity of the network. Then the emergence of the ping-pong effect inhibits the cooperative behavior and drives the system to vibrate between the globally cooperative and defective state finally. So in Fig. 4(b), the curves for  $r < 0.5$  ascend firstly and then descend to 0.5 gradually. While for  $r > 0.5$ , situations are rightly on the contrary.



**4. Further discussion on the PDG**

In the basic hypothesis of the PDG,  $P$  should be greater than  $S$ . The matrix in Eq. (3) should not induce qualitatively different characters if we set  $P = \varepsilon$  instead, with  $\varepsilon$  positive but significantly below unity [9]. When  $P = \varepsilon$ , Eq. (6) turns to

$$\rho_c = \frac{1}{1 + \exp\{k[\varepsilon - f(1 + \varepsilon - b)]/K\}}. \tag{10}$$

When  $k/K$  is large enough,  $\rho_c$  will be frozen near the value of zero. The ping-pong effect seems to disappear.

However, in Ref. [16], the authors suggested that the success of a strategy in the game should be measured by the ratio of total individual income and the number of neighbors. Eq. (2) is amended as

$$W_{i \rightarrow j} = \frac{1}{1 + \exp[(M_i/k_i - M_j/k_j)/K]}$$

with a relatively smaller  $K$ . Adopting this idea, we normalize the payoffs and the virtual payoffs in Eq. (4) as

$$W_i = \frac{1}{1 + \exp[(M_i/k_i - M'_i/k_i)/K]}$$

In this case, Eq. (10) turns to

$$\rho_c = \frac{1}{1 + \exp\{[\varepsilon - f(1 + \varepsilon - b)]/K\}}. \tag{11}$$

As the case in the above paragraphs, when the defective masses occur,  $\rho_c$  in Eq. (11) will not converge into a fixed value but plays a ping-pong switch between two states.

Simulation results are shown in Fig. 5, in which we can see the non-monotonic phenomena and the ping-pong effect still exist. Different from the case of  $P = S = 0$ , the ping-pong effect could not occur strictly

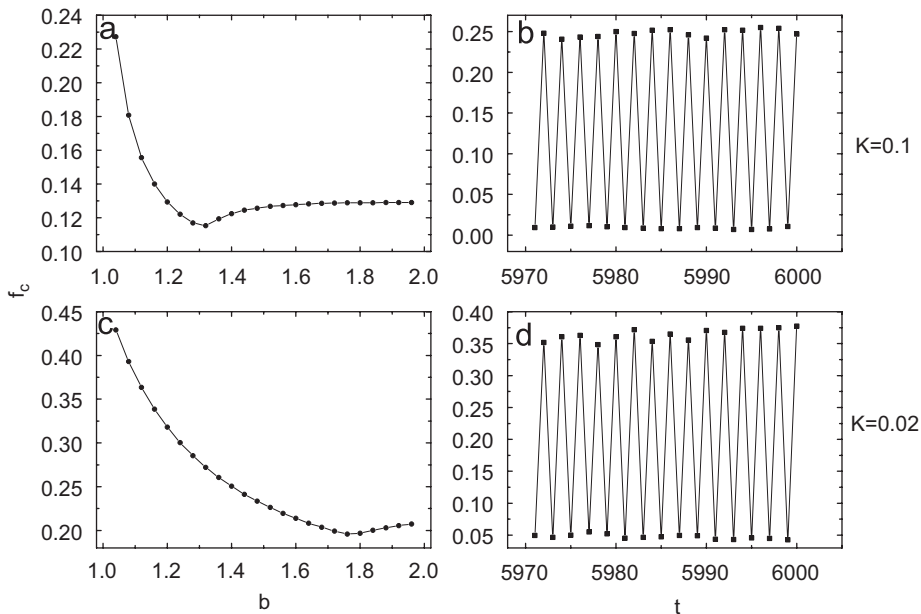


Fig. 5. (Color online) Cooperative frequency of the PDG for  $T = b, R = 1, P = 0.02, S = 0$ : (a) dependence of  $f_c$  on  $b$  when  $K = 0.1$ , (b) time series of  $f_c$  when  $K = 0.1$ , (c) dependence of  $f_c$  on  $b$  when  $K = 0.02$ , (d) time series of  $f_c$  when  $K = 0.1$ . The network parameter is  $N = 10000, m = 10$ .

between 0 and 0.5, but between two eclectic regions. Similar to the situation in Fig. 2(e), the ping-pong effect could not be globally coherent but still exists in the majority part of the system.

Therefore, the ping-pong effect is a reliable character in the PDG, rather than relying on an artificial setup of  $P = S = 0$ . None of our findings are qualitatively altered if we instead set  $P = \varepsilon$ , with  $\varepsilon$  positive but significantly below unity (so that  $T > R > P > S$  is strictly satisfied) [9].

## 5. Conclusion

In this paper, we have studied the evolutionary games on the scale-free network with a self-questioning updating mechanism. Numerical simulations and qualitative analyses are carried out on this model. Compared with the previous work in this field, our model does not have advantages in sustaining the cooperative behavior better and stabler, but shows interesting phenomena of non-monotony and discontinuous transition etc. These phenomena are related to the so-called “Cooperative Ping-pong Effect”, which has been observed earlier in the field of complex systems. In the evolutionary games, the ping-pong effect is driven by the players’ tendency of drifting with the tide. It happens under certain conditions in the PDG and almost everywhere in the SG. Note that in our games, although each player only pays attention to his own information of payoffs, the whole system exhibits highly self-organized characters and the players’ actions are highly synchronized. That means the payoffs of each player have contained plenty of information about the circumstances. In the evolutionary games, deciding according to yourself is as effective as learning from others. Furthermore, the self-questioning mechanism has avoided the system from being enmeshed in a trap of the globally defective state, which is a shortcoming for the previous models. Usually the ping-pong effect emerges when the situation is not so propitious to cooperation, thus it can improve the cooperative behavior in the system. However, large vibration, as well as defection, will also waste the resources seriously. So these problems are worthy of further studies, in order to find out more effective mechanisms to sustain the cooperative behavior better and to make use of the resources most efficiently.

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