Role of connectivity-induced weighted words in language games

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We present a modified naming game by introducing weights of words in the evolution process. We assign the weight of a word spoken by an agent according to its connectivity, which is a natural reflection of the agent's influence in population. A tunable parameter is introduced, governing the word weight based on the connectivity of agents. We consider the scale-free topology and concentrate on the efficiency of reaching the final consensus, which is of high importance in the self-organized system. Interestingly, it is found that there exists an optimal parameter value, leading to the fastest convergence. This indicates appropriate hub's effects favor the achievement of consensus. The evolution of distinct words helps to give a qualitative explanation of this phenomena. Similar nontrivial phenomena are observed in the total memory of agents with a peak in the middle range of parameter values. Other relevant characters are provided as well, including the time evolution of total memory and success rate for different parameter values as well as the average degree of the network, which are helpful for understanding the dynamics of the modified naming game in detail.

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The naming game has been considered as an important approach for understanding and characterizing the evolution of a language and more generally of a communication system without global supervision or a prior common knowledge [1,2]. It has been demonstrated that in these games, agents can achieve the consensus of naming an object through local pairwise interactions in a self-organized way, which can well explain the origin and evolution of languages [3–5]. Besides, such models were inspired by global coordination problems in artificial intelligence and peer-to-peer communication systems. A prototypical example is the so-called talking heads experiment [6], in which robots assign names to objects observed through cameras and negotiate these names with other agents. The naming game was also found meaningful for the new developed web tools [7,8], such as the del.icio.us or www.flickr.com, which enable web users to share classification of information in the web through tags invented by each user.

Recently, a minimal version of the naming game was proposed [9] with introduction of simple interaction rules together with the fully-connected relationship structure. This simplified game reproduced experimental phenomena and is easier for carrying out theoretical analyses. Further work has focused on the effects of low-dimensional lattices on the minimal naming game [10]. However, the rapid development of complex networks has revealed that real-world networks are neither fully connected graph nor lattices; instead, they share some common structural properties [11–17], such as the "small-world" [11] and the "scale-free" [12] features. Hence, there is a need to investigate the naming game on networks of these common features to better characterizing the evolution of languages or self-organized communication systems. As a result, several papers have focused on the influence of various topological properties on the corresponding dynamical behavior [18,19].

Previously reported results have shown that in the naming game, the system can reach a global consensus after a period of time. The specific time is a key measure for the convergence efficiency of the system, which is of practical importance. For instance, in social tagging sites, web sites or information tagged by different items will waste the resources and bring difficulties into communication. The fast consensus on tagging information is very beneficial for the information storage of web servers and information sharing of web users. Similarly, the fast collective agreement on naming objects plays a significant role in the cooperation among individuals for not only robots but also human beings. In this paper, we present a modified minimal naming game with respect to weighted words and embed the game on scale-free networks. We investigate the dynamics of the game influenced by the weight assignment and focus on the convergence time of reaching a global consensus. The words' weights are endowed according to the degrees of speakers and tuned by a model parameter. We found that there exist optimal parameter values, resulting in the fastest convergence of a shared lexicon. This nontrivial result indicates that strong influences of agents occupying high-degree nodes contribute significantly to the convergence efficiency of the system. On the other hand, too strong impacts of these individuals do harm to the consensus achievement. Besides, we study the memory length of agents and the success rate of negotiations as well for better understanding the evolution of the system.

Below, we describe the modified minimal naming game with weighted words. In the game, N identical agents observe a single object and try to communicate its name with others. Each agent is assigned an internal inventory or memory to store an unlimited number of different names or opinions. Initially, each agent has an empty memory. The evolutionary rules are as follows:

(i) at each time step, a hearer j is chosen at random and then the hearer chooses one of its neighbors as the speaker. It has been called reverse naming game. (In the reverse strat-

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egy, hub nodes have a higher probability of being selected as a speaker than in the normal direct strategy so that their influence is enhanced. Hence, the degree-based words' weights have more apparent influence on the dynamics of the naming game than the direct strategy.)

(ii) If the speaker i's memory (inventory) is empty, it invents a new name and records it together with assigning a weight k_i^α to the word, where k_i is the degree of node i and α is a tunable parameter. Otherwise, if i already knows one or more names of the object, the probability that it chooses one of them is proportional to the weight of the word. The invented or selected word is then transmitted to the hearer.

(iii) If the hearer j already has this transmitted name in its inventory, the negotiation is a success, and both agents preserve this name and cancel all other terms in their memory. Otherwise, the negotiation fails, and the new name is included in the memory of the hearer and no cancellation happens. No matter whether the negotiation is a success or not, the hearer j endows the retained or newly added word a weight k_i^α, where k_i is the degree of the speaker i. This assumption takes into account the fact that agents with more interactions usually cause more attraction in society, and hence words spoken from higher-degree agents may have stronger influences. Perhaps the power function of the speakers' degree is a natural reflection of the agents' influence, so that the words' weights can well capture the impact of agents on the dynamics of the system.

We need to point out that, though the word's weight has been considered previously in the naming game [20], our model is totally different. In previous models, the weight is determined by the success rate in negotiations [20], while in our model, we couple the word weight with topological properties to investigate the heterogeneous effect of the network on the evolutionary dynamics.

We consider the modified naming game on scale-free networks generated by using the simplest and well-known Barabási-Albert model [12]. Starting from m_0 nodes, each new node entering the network is allowed to establish m new links to existing nodes. Preferential attachment whereby an existing node i with a higher degree k_i has a higher probability $\Pi_i \sim k_i$ to attract a new link is imposed.

Relevant quantities in the study of the naming game are the total number of words in the system $N_w(t)$, which corresponds to the total memory used by agents, the average success rate $S(t)$ of the negotiation, and of high importance, the convergence time t_c. In Fig. 1, we report the evolution of total memory for different values of α and average degree $\langle k \rangle$ of networks. One can find that the maximum value of $N_w(t)$ shows a slight difference for positive values of α with fixing $\langle k \rangle$, while negative α induces lower $N_w(t)$. When α is fixed, the average degree $\langle k \rangle$ plays a crucial role in determining the maximum $N_w(t)$ with the $\langle k \rangle$ positively correlated with the maximum $N_w(t)$. Another behavior that should be noted is that larger total memory corresponds to the faster convergence of one name. In the original minimal naming game, the dependence of total memory on the average degree over several types of networks has been investigated [10,18,19]. The observed phenomena are qualitatively similar to our weighted generalization. Besides, one particular behavior found in Fig. 1 is that the convergence efficiency clearly depends on the values of parameter α, which governs the words' weight based on the speaker's degree. The effects of parameter values on reaching the collective consensus are studied in detail later.

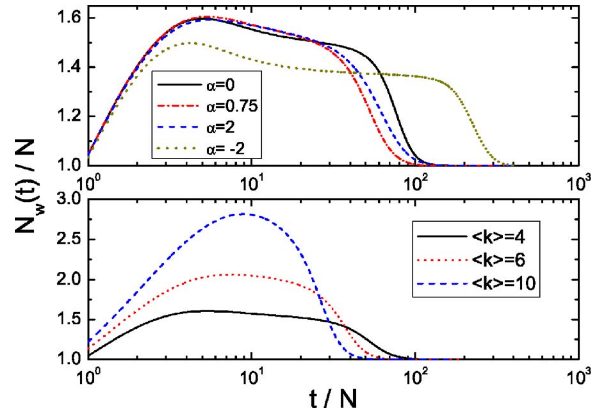

FIG. 1. (Color online) Evolution of the average memory per agent N_w/N versus rescaled time t/N for different values of α (top panel) with fixing average degree $\langle k \rangle$ to 4 and for different $\langle k \rangle$ with fixing $\alpha=0.75$. The network size is 5000. Each data point is obtained by averaging over 1000 runs and 10 network realizations.

The evolution of success rate $S(t)$ is displayed in Fig. 2. Similar to the cases of the unweighted naming game, after a quick increment, $S(t)$ reaches a plateau with slowly increasing velocity. All of a sudden, $S(t)$ increases towards 1 over very short periods. For different $\langle k \rangle$ with fixed α, the higher values of $\langle k \rangle$, the lower values of the plateau, and the earlier of the sudden increase. Our results are consistent with the mean-field case [18], which can be considered as the large limit of $\langle k \rangle$. The success rate shows as well the same qualitative results as the original minimal naming game [18]. While for the identical $\langle k \rangle$ with different α, the words' weights almost have no effect on the plateau as well as the initial evolution, but contribute to the beginning of the sudden increase and time to approach 1. The behavior of $S(t)$

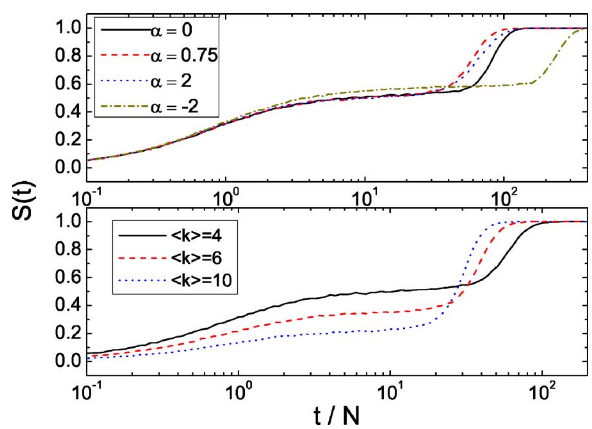

FIG. 2. (Color online). Evolution of average success rate $S(t)$ for different α with keeping $\langle k \rangle=4$ (top) and for different $\langle k \rangle$ with fixing α. Other parameter values and averaging are the same as in Fig. 1.

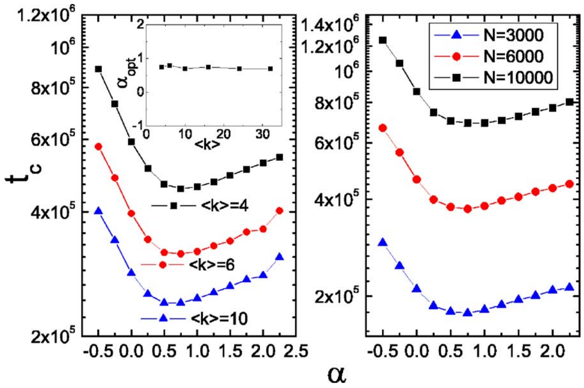

FIG. 3. (Color online). Left panel: convergence time t_c vs α for different $\langle k\rangle$ with $N=5000$. The inset reports the optimal values α_{opt} corresponding to the minimum t_c. Right panel: t_c vs α for different network size N with $\langle k\rangle=6$. Results are averaged over a large number of runs and network realizations.

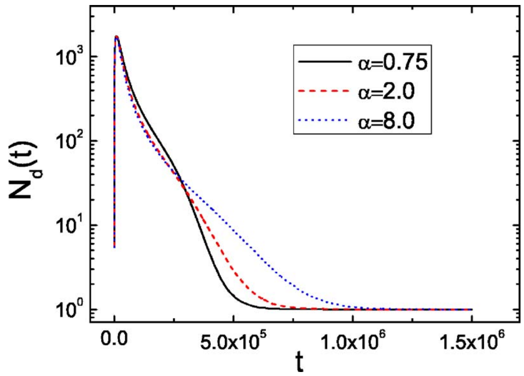

FIG. 4. (Color online) Evolution of the number of different words $N_d(t)$ in the system for different α with fixing $\langle k\rangle=4$. The network size is 5000.

agrees with the convergence efficiency that the faster $S(t)$ approaches 1, the shorter the convergence time takes. Moreover, compared with Fig. 1, the fast increment of the success rate corresponds to the fast reduction of the total memory of agents.

Next we turn to the most important quantity, convergence time t_c, affected by the assignment of words' weights. As shown in Fig. 3, interesting results are observed, that is, there exists an optimal value of α, resulting in the fastest final consensus. The value is almost independent of the average degree and the network size, as shown in the left and right panels of Fig. 3. The enhancement of convergence efficiency in the weighted case can be easily understood by noting the fact that the degree distribution of scale-free networks is highly heterogenous. High-degree nodes take the minority and have more chances to be selected as speakers in the reverse naming game. Hence, the words spoken by agents of high degree are more easily spread in the system and retained of high probability, consequently favors the achievement of the final consensus. However, contrary to one's intuition, very strong influences of high-degree agents play a negative role, even worse than the case without weights. Scale-free networks usually have a few hubs while most nodes are of very low degree. Very high values of α correspond to the authority of those hubs, and only the words invented by hubs diffuse in the system. Hence, there will exist some word clusters around a few hubs, within which agents share a word invented by the hub. Then the competition of word clusters emerges, leading to longer convergence time. (We have checked that if the direct naming game strategy is adopted, the optimal α moves towards a larger value, approximately 1.5. In this case, the influence of hubs is inhibited, so that compared to the reverse strategy the words spreading from those hubs require stronger weights to speed up the convergence. Thus, the optimal α increases.)

The competition among word clusters can be partially reflected by the evolution of the number of different words $N_d(t)$. As displayed in Fig. 4, initially, the invented different words increase sharply. After that, a sudden decrease occurs and higher values of α lead to faster decrease velocity. At this stage, word clusters emerge. As discussed above, a stronger influence of hubs, i.e., larger α promotes the formation of clusters, reflected by the more quick reduction of $N_d(t)$. When only a few different words remain in the system, for example, $N_d(t)<10$, in the case of larger α, the system takes a much longer time to reach the final consensus. This indicates the competition of small numbers of big word clusters, and the biggest cluster needs a very long time to invade others and dominate the system.

Whereafter, we study the normalized maximum total memory of agent N_w^{max} as a function of parameter α for different average degree $\langle k\rangle$ (Fig. 5). The normalization is performed by dividing N_w^{max} with the highest value for a given $\langle k\rangle$. One can find that there exist as well nonmonotonous behaviors with a peak in the middle range of α. In contrast to the convergence time, the maximum values of N_w^{max} depend on $\langle k\rangle$ with a move towards a lower value of α as $\langle k\rangle$ increases. For large α, the fast construction of word clusters quickly decreases the memory capacity of agents, resulting in low values of N_w^{max}. On the other hand, in the case of negative α, the hub effect in the reverse naming game is strongly inhibited, so that the system needs a long time to achieve the final agreement, as shown in Fig. 1. Moreover, like previously reported results, the total memory of agent is usually negatively correlated with convergence time. There-

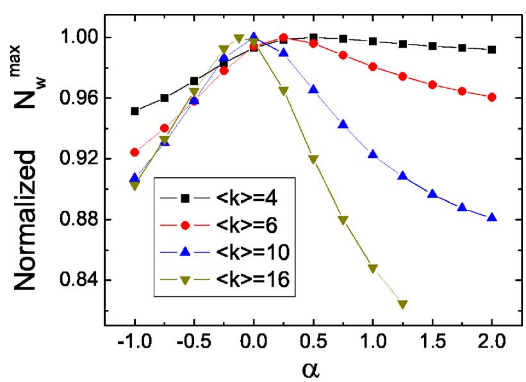

FIG. 5. (Color online) Normalized maximum total memory used by agent N_w^{max} as a function of α for different $\langle k\rangle$. The network size is 5000. The averaging is the same as in Fig. 1.

fore, there should exist a peak value of N_w^{max} in the middle range of α by combining the situations of both low and high values of α. The movement of the specific α corresponding to the maximum N_w^{max} is attributed to the strong effect of the average degree on the total memory, such as the results displayed in Fig. 1.

In conclusion, we have studied the modified minimal naming game with respect to words' weights on the scale-free network. The weight assignment is made according to the influence of speakers characterized by the power function of their corresponding degrees, where the exponent is a tunable parameter, governing the word's weight. We have reported the dynamical behavior of statistical quantities in the naming game for different parameter values, including the total memory (inventory) of agents, success rate of negotiations, and the number of total different invented words. Of particular importance, we focus on the convergence efficiency of the system. Interestingly, we found some nonmonotonous behaviors in the consensus achievement with a specific value of α leading to the shortest convergence time. This finding is of both scientific interests and practical importance. The existence of the optimal parameter is explained qualitatively by means of the formation of word clusters. Furthermore, we have investigated the maximum total memory depending on the parameter and observed as well the nonmonotonous behavior with a peak in the middle range of parameter values. Our idea of the weighted naming game by considering the coupling of the topology and the words' weights may gain some attention in the field of language games, since our model generates some interesting results, which is as well of practical significance.

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