Mutual selection model for weighted networks

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For most networks, the connection between two nodes is the result of their mutual affinity and attachment. In this paper, we propose a mutual selection model to characterize the weighted networks. By introducing a general mechanism of mutual selection, the model can produce power-law distributions of degree, weight, and strength, as confirmed in many real networks. Moreover, we also obtained the nontrivial clustering coefficient C, degree assortativity coefficient r, and degree-strength correlation depending on a single parameter m. These results are supported by present empirical evidence. Studying the degree-dependent average clustering coefficient $C(k)$ and the degree-dependent average nearest neighbors' degree $k_{nn}(k)$ also provide us with a better description of the hierarchies and organizational architecture of weighted networks.

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I. INTRODUCTION

In the past few years, physicists have been greatly devoted to understanding and characterizing the underlying mechanisms of complex networks, e.g., the Internet [1], the World Wide Web (WWW) [2], the scientific collaboration networks (SCN) [3–5], and the world-wide airport networks (WAN) [6–8]. Until now, network researchers have mainly focused on the topological aspect of graphs, that is, unweighted networks. Typically, Barabási and Albert proposed a famous model (BA model) that introduces the linear degree preferential attachment mechanism to study unweighted growing networks [9–11]. However, this model is still insufficient to describe real networks' structure if considering the properties of clustering coefficient and assortive mixing. The hypothesis of a linear attachment rate is empirically supported by measuring different real networks, but the origin of the ubiquity of the linear preferential attachment is not clear yet. Recently, the availability of more complete empirical data has allowed scientists to consider the variation of the weights of links that reflect the physical characteristics of many real networks. It is well-known that networks are not only specified by their topology but also by the dynamics of weight taking place along the links. For instance, the heterogeneity in the intensity of connections may be very important in understanding network systems. Traffic amount characterizing the connections of communication systems or large transport infrastructure is fundamental for a full description of these networks. Take the WAN for example: each given edge weight w_{ij} (traffic) is the number of available seats on direct flight connections between the airports i and j. In the SCN, the nodes are identified with authors and the weight depends on the number of coauthored papers. Obviously, there is a tendency of modeling complex networks that goes beyond the purely topological point of view, and investigating how the weight distribution affects the dynamics upon networks. In the light of this need, Barrat *et al.* presented a model (BBV model) that integrates the topology and weight dynamical evolution to study the growth of weighted networks [12–14]. Their model yields scale-free properties of the degree, weight, and strength distributions, controlled by an introduced parameter δ. However, its weight dynamical evolution is triggered only by newly added vertices, hardly resulting in satisfying interpretations to the collaboration networks or the airport systems.

The properties of a graph can be expressed via its adjacency matrix a_{ij}, whose elements take the value 1 if an edge connects the vertex i to the vertex j, and 0 otherwise. The data contained in the previous data sets permit one to go beyond this topological representation by defining a weighted graph. A weighted network is often described by a weighted adjacency matrix w_{ij}, which represents the weight on the edge connecting vertices i and j, with $i,j=1,\ldots,N$, where N is the size of the network. We will only consider undirected graphs, where the weights are symmetric ($w_{ij}=w_{ji}$). As confirmed by measurements, complex networks often exhibit a scale-free degree distribution $P(k)k^{-\gamma}$, with $2\leq\gamma\leq 3$ [6,7]. The weight distribution $P(w)$ that any given edge has weight w is another significant characterization of weighted networks, and it is found to be heavy tailed, spanning several orders of magnitude [15]. A natural generalization of connectivity in the case of weighted networks is the vertex strength described as $s_i=\Sigma_{j\in\Gamma(i)}w_{ij}$, where the sum runs over the set $\Gamma(i)$ of neighbors of node i. The strength of a vertex integrates the information about its connectivity and the weights of its links. Take the WAN for example: the strength represents the actual traffic going through a vertex and the measure of the size and importance of each airport is obvious. For the SCN, the strength is a measure of scientific productivity, since it is equal to the total number of publications of any given scientist. This quantity is a natural measure of the importance or centrality of a vertex in the network. Empirical evidence indicates that in most cases the strength distribution has a fat tail [7], similar to the power law of degree distribution. Highly correlated with the degree, the strength usually displays scale-free property $s\sim k^{\beta}$ [16–20].

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The previous models of complex networks always incorporate the (degree or strength) preferential attachment mechanism, which may result in scale-free properties. Essentially speaking, this mechanism just describes interactions between the newly added node and the old ones. The fact is that such interactions also exist among old nodes. This perspective has been practiced in the work of Dorogovtsev and Mendes (DM) [21], who proposed a class of undirected and unweighted models where new edges are added between old sites (internal edges) and existing edges can be removed (edge removal). On the other hand, we argue that any connection is a result of mutual affinity and attachment between nodes, while many network models seem to ignore this point. Traditional models often present us such an evolution picture: pre-existing nodes are passively attached by newly adding nodes according to linear degree (or strength) preferential mechanism. This picture is just a partial aspect for most complex networks. It is worth nothing that the creation and reinforcement of internal connections is an important aspect for understanding real graphs [22].

In this paper, we shall present a model for weighted networks that considers the topological evolution under the general mechanism of mutual selection and attachment between vertices. It can mimic the reinforcement of internal connections and the evolution of many infrastructure networks. The diversity of scale-free characteristics, nontrivial clustering coefficient, assortativity coefficient, nonlinear strength-degree correlation, and hierarchical structure that have been empirically observed can be well explained by our microscopic mechanisms. Moreover, in contrast with previous models where weights are assigned statically [23,24] or rearranged locally [12], we allow weights to be widely updated.

II. THE MUTUAL SELECTION MODEL

The model starts from an initial configuration of N_0 vertices fully connected by links with assigned weight w_0. The model is defined on two coupled mechanisms: the topological growth and the mutual selection dynamics

A. Topological growth

At each time step, a new vertex is added with n edges connected to n previously existing vertices, choosing preferentially nodes with large strength; i.e., a node i is chosen according to the strength preferential probability

$$\Pi_{new\to i} = \frac{s_i}{\sum_k s_k}. \tag{1}$$

The weight of each new edge is also fixed to w_0.

B. Mutual selection dynamics

According to the probability

$$\Pi_{i\to j} = \frac{s_j}{\sum_k s_k - s_i}, \tag{2}$$

each existing node i selects m other old nodes for potential interaction. If a pair of unlinked nodes is mutually selected, then an internal connection will be built between them. Or, if two connected nodes select each other, then their existing connection will be strengthened; i.e., their edge weight will be increased by w_0. Mutual selection means that the interaction between components i and j is due to their common choice and attachment. Here, the parameter m is the number of candidate vertexes for creating or strengthening connections. Later, we will see that m also controls the growing speed of the network's total strength, for example, the increasing rate of total information in a communication system. Remark: considering the normalization requirement and that vertices are not permitted to connect themselves, the denominator of $\Pi_{i\to j}$ contains the term $-s_i$.

We argue that connections in most real networks are due to the mutual selections and attachments between nodes. Take the SCN for example: collaboration among scientists requires their common interest and mutual acknowledgments. Unilateral effort does not promise collaboration. Two scientists with strong scientific potentials (large strengths) and long collaborating history are more likely to publish papers together during a certain period. Likewise, for the Movie Actor Collaboration Networks (MACN), two actors that both have high popularity are more likely to boost up the box office if they costar. So, it is reasonable to assume that each node is more likely to choose those nodes with large strength when building or strengthening connections. This also indicates that pre-existing nodes with large strength will not be passively attached by nodes with small strength. There is competition and adaptation in such complex systems. Both natural and social networks bear such a property or mechanism during their evolutions. The above description of our model also could satisfactorily explain the WAN. The weight here denotes the relative magnitude of the traffic on a flight connection. At the beginning of the airport network construction, the air line is usually built between metropolises with high status in both economy and politics. Once a new air line is created between two airports, it will trigger more intense traffic activities depending on the specific nature of the network topology and the microdynamics. Due to the improvement of national economy and the expansion of population, the air traffic between metropolises will increase. There is an obvious need for other cities to build new airports to connect the metropolises for their great importance. Indeed, it is reasonable that the traffic between metropolises will grow faster than that between other cities, each of which possesses lower economical and political status and a smaller population who can afford airplane tickets. But, due to the limit of energy and resources, each node can only afford a limited number of connections. Hence facing the vertex pool, they have to choose. Take the WAN for example: an airport cannot afford the cost of connecting all the other airports.

The network provides the substrate on which numerous dynamical processes occur. Technological networks provide a large empirical database that simultaneously captures the topology and the dynamics taking place on it. For the Internet, the information flow between routers (nodes) can be represented by the corresponding edge weight. The total information load that each router deals with can be denoted by the node strength, which also represents the importance of a given router. The increasing information flow as an internal demand always spurs the expansion of technological networks. Specifically, the largest contribution to the growth is given by the emergence of links between already existing nodes. This clearly points out that the Internet growth is strongly driven by the need for a redundancy wiring and an increasing need of available bandwidth for data transmission [17]. On one hand, newly built links (between existing routers) are supposed to preferentially connect high-strength routers; otherwise, it would lead to unexpected traffic congestion along indirect paths that connect those high-strength nodes. Naturally, information traffic along existing links between high-strength routers, in general, increases faster than that between low-strength routers. This phenomenon also could be reproduced in our model. On the other hand, new routers preferentially connect to routers with larger bandwidth and traffic handling capabilities (the strength-driven attachment). This characteristic also exists in an airport system, power grid, and railroad network, and they could be explained by our mechanisms.

III. PROBABILITY DISTRIBUTIONS AND STRENGTH EVOLUTION

The network growth starts from an initial seed of N_0 nodes, and continues with the addition of one node per unit time, until a size N is reached. Hence, the model time is measured with respect to the number of nodes added to the graph, i.e., $t=N-N_0$, and the natural time scale of the model dynamics is the network size N. Using the continuous approximation, we can treat k, w, s, and the time t as continuous variables [1,9]. Then, the edge weight w_{ij} is updated according to this evolution equation:

$$\frac{dw_{ij}}{dt} = m\frac{s_j}{\sum_k s_k - s_i} \times m\frac{s_i}{\sum_k s_k - s_j} = \frac{m^2 s_i s_j}{(\sum_k s_k - s_j)(\sum_k s_k - s_i)}. \tag{3}$$

There are two processes that contribute to the increment of strength s_i. One is the creation or reinforcement of internal connections incident with node i, the other is the attachment to i by newly added node. So, the rate equation of strength i can be written as below:

$$\frac{ds_i}{dt} = \sum_j \frac{dw_{ij}}{dt} + n \times \frac{s_i}{\sum_k s_k} \approx \frac{m^2 s_i}{\sum_k s_k}\frac{\sum_j s_j}{\sum_k s_k} + \frac{n s_i}{\sum_k s_k} = (m^2+n)\frac{s_i}{\sum_k s_k}. \tag{4}$$

This equation may be written in a more compact form by noticing that

$$\sum_{i=1}^{t} s_i = \int_0^t \sum_{k\in\Lambda} \frac{ds_k}{dt}dt + 2nt \approx (m^2+2n)t, \tag{5}$$

where Λ represents the set of existing nodes at time step t. By plugging this result into Eq. (4), we obtain the following strength dynamical equation:

$$\frac{ds_i}{dt} = \frac{m^2+n}{m^2+2n}\frac{s_i}{t}, \tag{6}$$

which can be readily integrated with initial conditions $s_i(t=i)=n$, yielding

$$s_i(t) = n\left(\frac{t}{i}\right)^{(m^2+n)/(m^2+2n)}. \tag{7}$$

The equation $\sum_i s_i \approx (m^2+2n)t$ also indicates that the total strength of the vertices in the statistical sense is uniformly increased with the size of network. As one see, can the growing speed of the network's total strength load is mainly determined by the model parameter m.

The knowledge of the time evolution of the various quantities allows us to compute their statistical properties. Indeed, the time $t_i=t$ at which the node i enters the network is uniformly distributed in $[0,t]$ and the degree probability distribution can be written as

$$P(s,t) = \frac{1}{t+N_0}\int_0^t \delta(s - s_i(t))dt_i, \tag{8}$$

where $\delta(x)$ is the Dirac delta function. Using the equation $s_i(t) \sim (t/i)^{\theta})$ obtained from Eq. (7), one obtains in the infinite size limit $t\to\infty$ the distribution $P(s)\sim s^{\alpha}$ with $\alpha=1+1/\theta$

$$\alpha = 2 + n/(m^2+n). \tag{9}$$

Obviously, when $m=0$ the model is topologically equivalent to the BA network and the value $\alpha=3$ is recovered. For larger values of m, the distribution is gradually getting broader with $\alpha\to 2$ when $m\to\infty$.

We performed numerical simulations of networks generated by choosing different values of m and fixing $n=5$ and $w_0=1$. Considering that every vertex strength can at most increase by m from internal connections, and a newly added node can connect with no more than n existing nodes, it is easy to conclude that the initial network configuration must satisfy $N_0 \geqslant \max(m+1,n)$. For example, if $m=10$, then $N_0 \geqslant 11$. In the following simulations, we will simply take $N_0=\max(m+1,n)$. We have checked that the scale-free properties of our model networks are independent of the initial conditions. Numerical simulations are consistent with our theoretical predictions, which verify again the reliability of our present results. Figure 1 gives the probability distribution $P(s)\sim s^{\alpha}$, which is in excellent agreement with the theoretical predictions. In Fig. 2 we show the behavior of the vertices' strength versus time for different values of m, recovering the behavior predicted by analytical methods. We also report the average strength s_i of vertices with degree k_i, which displays a nontrivial power-law behavior $s\sim k^{\beta}$ as confirmed by empirical measurement. Unlike BBV networks (where $\beta=1$),

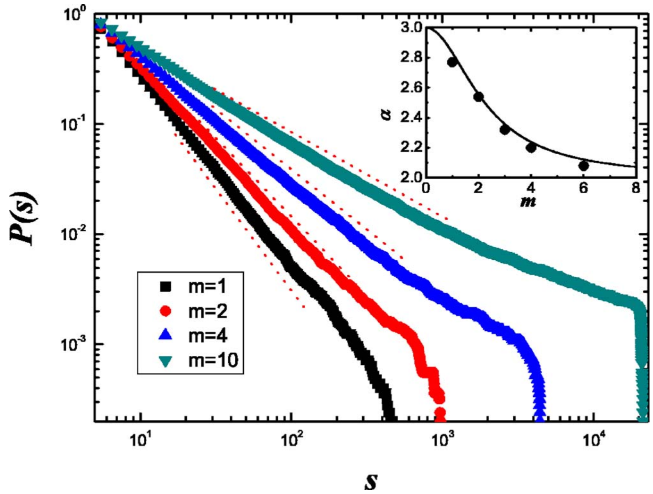

FIG. 1. (Color online) Probability distribution $P(s)$. Data are consistent with a power-law behavior $s^{-\alpha}$. In the inset we give the value of α obtained by data fitting (filled circles), together with the analytical expression $\alpha=2+n/(m^2+n)=2+5/(m^2+5)$ (line). The data are averaged over ten independent runs of network size $N=5000$.

the exponent β here varies with the parameter m in a nontrivial way as shown in Fig. 3. The nontrivial $s\sim k^{\beta}$ correlation demonstrates the phenomenon of "rich gets richer" conformed by real observation. More importantly, one could check the scale-free property of degree distribution $[P(k)\sim k^{-\gamma}]$ by combining $s\sim k^{\beta}$ with $P(s)\sim s^{-\alpha}$. Considering the conservation of probability

$$\int_0^{\infty} P(k)dk = \int_0^{\infty} P(s)ds, \qquad (10)$$

we can easily calculate the exponent γ

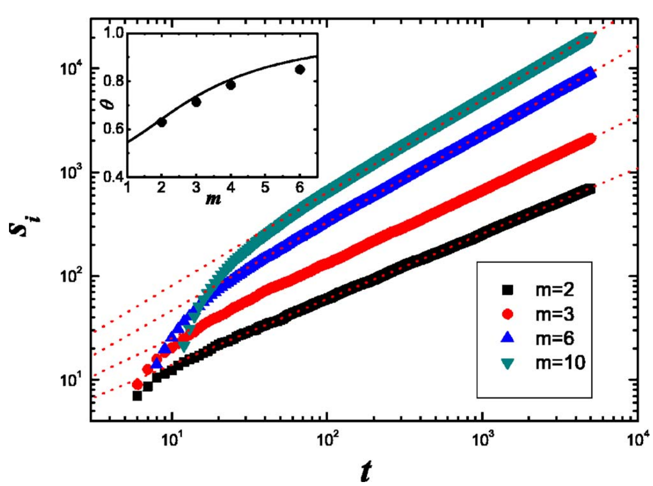

FIG. 2. (Color online) Evolution of strength of vertices during the growth of network for various of m. In the inset we give the value of θ obtained by data fitting (filled circles), together with the analytical expression $\theta=(m^2+n)/(m^2+2n)=(m^2+5)/(m^2+10)$ (line).

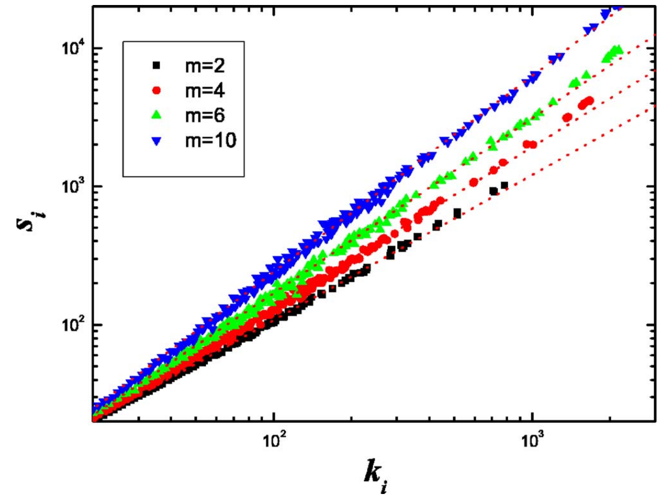

FIG. 3. (Color online) Strength s_i versus k_i for different m (log-log scale). Linear data fitting gives slope 1.04, 1.16, 1.26, and 1.41 (from bottom to top), demonstrating the correlation of $s\sim k^{\beta}$.

$$P(k) = P(s)\frac{ds}{dk} = s^{-\alpha}\beta k^{\beta-1} = \beta k^{-[\beta(\alpha-1)+1]}, \qquad (11)$$

giving $\gamma=\beta(\alpha-1)+1$. The scale-free properties of degree obtained from simulations are presented in Fig. 4. Together, the power-law distribution of weight $P(w)$ (implying the probability of finding a link with weight w) is shown in Fig. 5. The simulation consistency of scale-free properties indicates that our model can indeed produce power-law distributions of degree, weight, and strength. In this case, the numerical simulations of the model reproduce the behaviors predicted by the analytical calculations.

IV. CLUSTERING AND CORRELATION

Many real networks in nature and society share two generic properties: scale-free distributions and high degree of

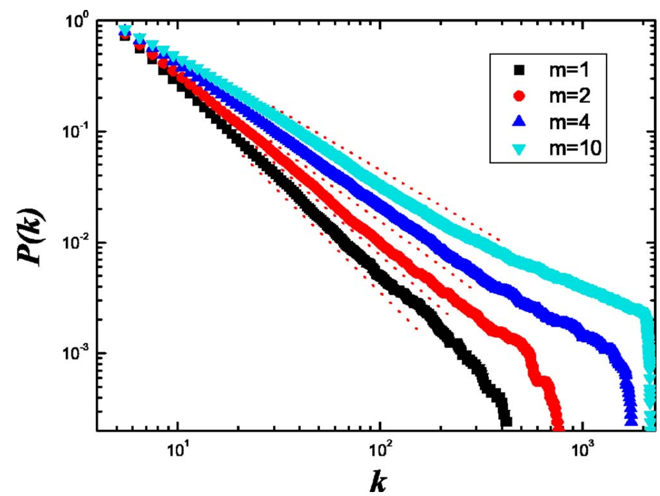

FIG. 4. (Color online) Probability distribution of the degrees $P(k)\sim k^{-\gamma}$ for different m. The data are averaged over ten independent runs of network size $N=5000$.

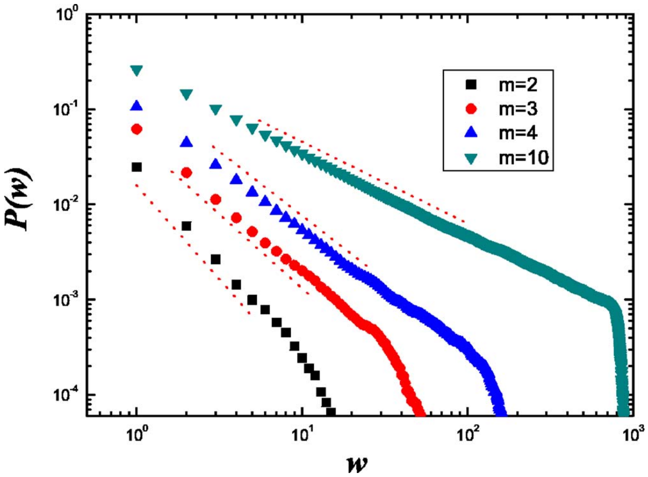

FIG. 5. (Color online) Probability distribution of the weights $P(w) \sim w^{-\eta}$ for various m. The data are averaged over ten independent runs of network size $N=5000$.

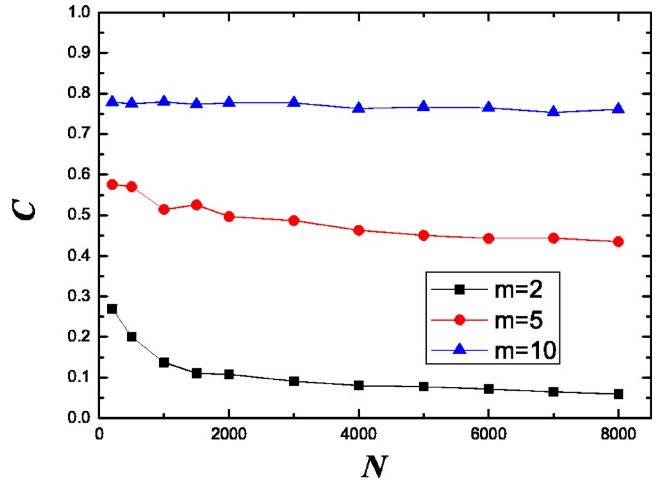

FIG. 6. (Color online) The evolution of clustering coefficient (or C versus N) which converges soon.

clustering. Along with the general vertices hierarchy imposed by the scale-free strength distribution, complex networks show an architecture imposed by the structural and administrative organization of these systems, which is mathematically encoded in the various correlations existing among the properties of different vertices. For this reason, a set of topological and weighted quantities is usually studied in order to uncover the network architecture. The first and widely used quantity is given by the *clustering* of vertices. The clustering of a vertex i is defined as

$$c_i = \frac{1}{k_i(k_i-1)} \sum_{j,h} a_{ij} a_{ih} a_{jh}, \qquad (12)$$

and measures the local cohesiveness of the network in the neighborhood of the vertex. Indeed, it yields the proportion of interconnected neighbors of a given vertex. The average over all vertices gives the network *clustering coefficient*, which describes the statistics of the density of connected triples. Further information can be gathered by inspecting the average clustering coefficient $C(j)$ restricted to classes of vertices with degree k

$$C(k) = \frac{1}{NP(k)} \sum_{i,k_i=k} c_i. \qquad (13)$$

In many networks, the average clustering coefficient $C(k)$ exhibits a highly nontrivial behavior with a power-law decay as a function of k [19], indicating that low-degree nodes generally belong to well-interconnected communities (high clustering coefficient), while high-degree sites are linked to many nodes that may belong to different groups which are not directly connected (small clustering coefficient). This is generally the feature of a nontrivial architecture in which hubs (high degree vertices) play a distinct role in the network. Numerical simulations indicate that for large m, the clustering coefficient $C(N)$ is almost independent of N (as we can see in Fig. 6), which agrees with the finding in several real networks [9]. Generally, when the network size N is larger than 5000, the clustering coefficient is nearly stable. So, most computer runs are assigned 5000. Still, it is worth noting that for the BA networks, $C(N)$ is nearly zero, far from the practical nets that exhibit a variety of small-world properties. In the present model, however, clustering coefficient C is fortunately found to be a function of m (see Fig. 7), also supported by empirical data in a broad range.

Finally, the clustering coefficient $C(k)$ depending on connectivity k for increasing m is also interesting and shown in Fig. 8. For clarity, we add the dashed line with slope -1 in the log-log scale. These simulation results are supported by recent empirical measurements in many real networks. For the convenience of comparison with Fig. 8, we use two figures from Ref. [19] as our Fig. 9, from which one can see the agreement between simulation results of clustering-degree correlation and empirical evidence is quite excellent. Though some previous models [25,26] can generate the power-law decay of the clustering-degree correlation, none of them as far as we know can produce the flat head as found in real

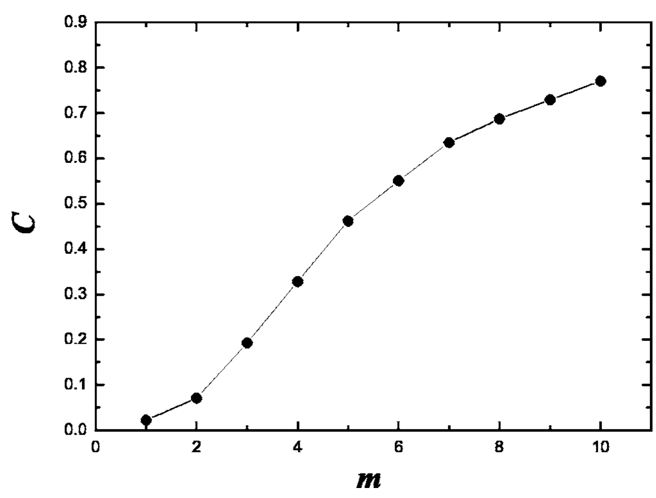

FIG. 7. Clustering coefficient C depending on the parameter m.

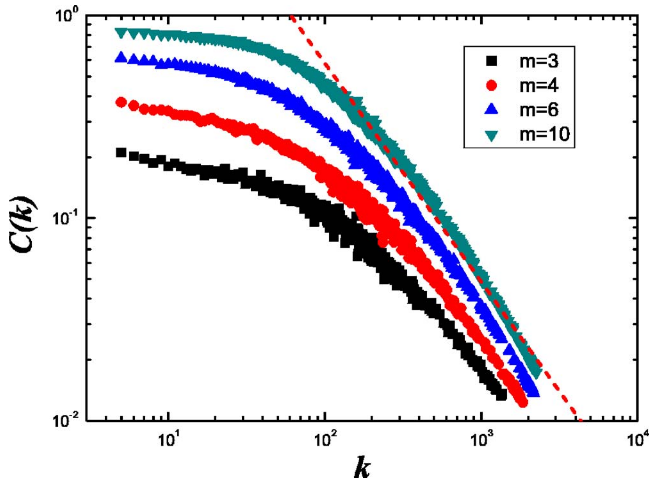

FIG. 8. (Color online) The clustering coefficient $C(k)$ depending on connectivity k for increasing m. For comparison, the dashed line has slope -1 in the log-log scale.

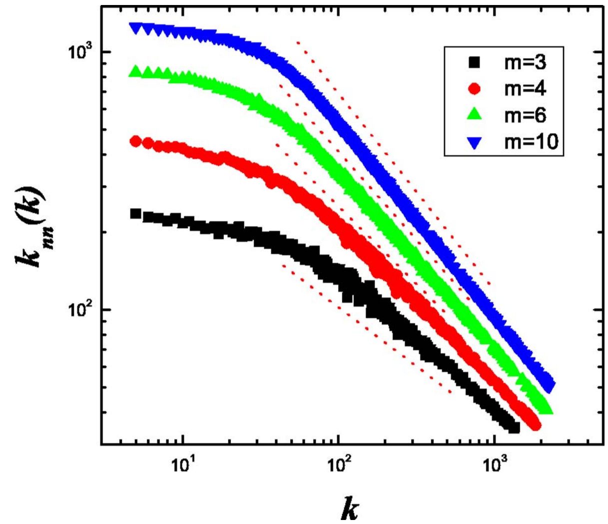

FIG. 10. (Color online) Average connectivity $k_{nn}(k)$ of the nearest neighbors of a node depending on its connectivity k for different m.

graphs. This is a special property that our model successfully obeys.

Another important source of information is the degree correlation of vertex i and its neighbor. The *average nearest-neighbor degree* is proposed to measure these correlations

$$k_{nn,i} = \frac{1}{k_i}\sum_j a_{ij}k_j. \tag{14}$$

Once averaged over classes of vertices with connectivity k, the average nearest-neighbor degree can be expressed as

$$k_{nn}(k) = \sum_{k'} P(k'|k), \tag{15}$$

providing a probe on the degree correlation function. Here, $P(k'|k)$ denotes the conditional probability that a k-degree vertex connects to a k'-degree neighboring vertex. If degrees of neighboring vertices are uncorrelated, $P(k'|k)$ is only a function of k' and thus $k_{nn}(k)$ is a constant. When correlations are present, two main classes of possible correlations have been identified: *assortative* behavior if $k_{nn}(k)$ increases with k, which indicates that large degree vertices are preferentially connected with other large degree vertices, and *disassortative* if $k_{nn}(k)$ decreases with k, which denotes that links are more easily built between large degree vertices and small ones. The above quantities provide clear signals of a structural organization of networks in which different degree classes show different properties in the local connectivity structure. In light of this measure, we also perform computer simulations to test the $k_{nn}(k)-k$ correlation, as shown in Fig. 10. As $k_{nn}(k)$ decreases with k, one may find that our model can best illustrate disassortative networks in reality, i.e., technological networks (e.g., Internet, WAN) and biological networks (e.g., protein folding networks). As for the social networks, connections among people may be assortative by language or by race. Newman proposed some simpler measures to describe these types of mixing, which we call assortativity coefficients [27]. Almost all the social networks studied show positive assortativity coefficients while all the others, including technological and biological networks, show negative coefficients. It is not clear if this is a universal property; the origin of this difference is not understood ei-

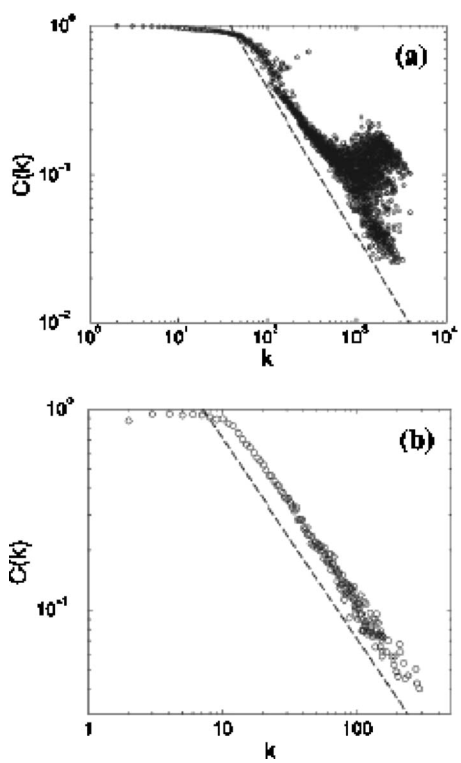

FIG. 9. The scaling of $C(k)$ with k for two real networks [19]: (a) Actor network, two actors being connected if they acted in the same movie according to the www.IMDB.com database. (b) The semantic web, connecting two English words if they are listed as synonyms in the Merriam Webster Dictionary. The dashed line in each figure has slope -1.

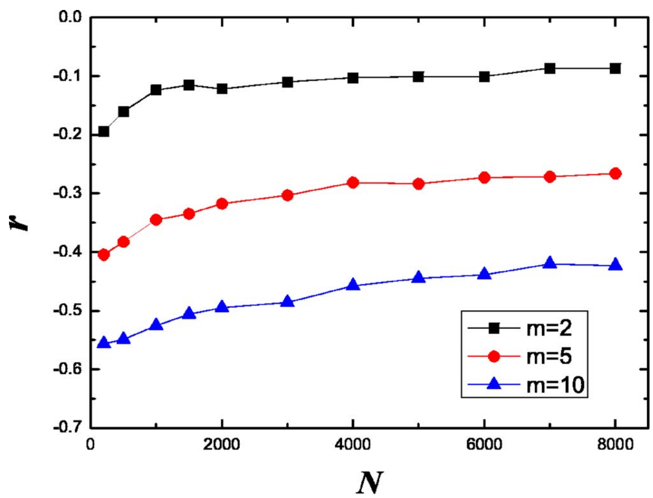

FIG. 11. (Color online) Degree-degree correlation r depending on N. The evolution of r converges soon.

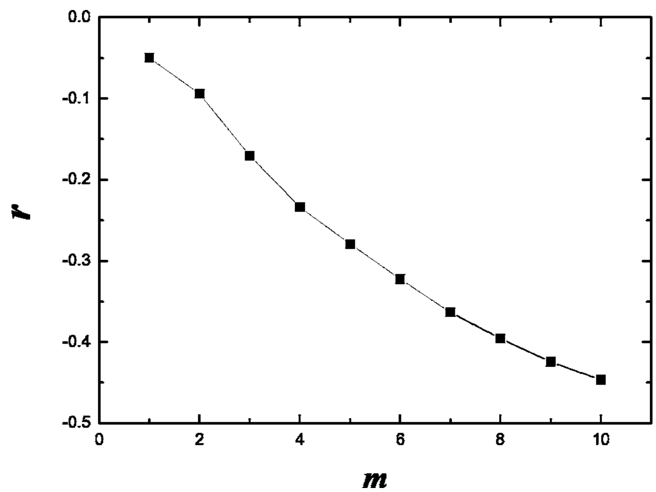

FIG. 12. Degree-degree correlation r depending on m.

ther. In our view, it represents a feature that should be addressed in each network type individually. In the following, we use the formula proposed by Newman in Ref. [27]:

$$r = \frac{M^{-1}\sum_i j_i k_i - \left[M^{-1}\sum_i \frac{1}{2}(j_i + k_i)\right]^2}{M^{-1}\sum_i \frac{1}{2}(j_i^2 + k_i^2) - \left[M^{-1}\sum_i \frac{1}{2}(j_i + k_i)\right]^2}, \qquad (16)$$

where j_i, k_i are the degrees of vertices at the ends of the ith edges, with $i=1,\ldots,M$ (M is the total number of edges in the observed graph). We calculate the degree assortativity coefficient (or degree-degree correlation) r of the graphs generated by our model. For large N (e.g., $N>5000$), the degree-degree correlation r is almost independent of the network size (see Fig. 11). Simulations of r depending on m are given in Fig. 12 and supported by empirical measurements for disassortative networks [27].

V. CONCLUSION AND OUTLOOK

In sum, integrating the mutual selection mechanism between nodes and the growth of strength preferential attachment, our network model provides a wide variety of scale-free behaviors, tunable clustering coefficient, and nontrivial (degree-degree and strength-degree) correlations, just depending on the parameter m which governs the total weight growth. All the results of network properties are found to be supported by various empirical data. Interestingly and specially, studying the degree-dependent average clustering coefficient $C(k)$ and the degree-dependent average nearest-neighbors' degree $k_{nn}(k)$ also provides us with a better description of the hierarchies and organizational architecture of weighted networks. Our model may be very beneficial for future understanding or characterizing real networks. Though our model can just produce disassortative networks (most suitable for technological and biological ones), which is one of its limitations, we always expect some model versions or variations that generate weighted networks with assortative property. Due to the apparent simplicity of our model and the variety of tunable results, we believe that some of its extensions will probably help address (e.g., social) networks. Therefore, we believe our present model, for all practical purposes, might demonstrate its application in future weighted network research.

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