Feedback reciprocity mechanism promotes the cooperation of highly clustered scale-free networks

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We study how the clustering coefficient influences the evolution of cooperation in scale-free public goods games. In games played by groups of individuals, triangle loops provide stronger support for mutual cooperation to resist invasion of selfish behavior than that in the absence of such loops, so that diffusion of cooperative behavior is relatively promoted. The feedback reciprocity mechanism of triangle plays a key role in facilitating cooperation in high clustered networks.

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Evolutionary game theory provides a theoretical framework to understand the evolution and maintenance of ubiquitous cooperation in biological, economic and social systems [1]. In order to depict interactions among selfish individuals, some meaningful game models are proposed, where the public goods game (PGG) as the multiperson prisoner's dilemma game is a common paradigm to characterize group interactions [2]. In a typical PGG, all individuals can choose to cooperate or defect. Cooperators contribute an amount c to the public goods game, while defectors invest nothing. The total contribution is multiplied by a factor r, and redistributed uniformly among all players. It has been known that for $r<N$, defectors will dominate the whole population which results in the *tragedy of the commons* and the *free rider problem*.

To escape the dilemma, a variety of mechanisms favoring cooperation have been proposed, such as reward and punishment [3], voluntary participation [4,5], etc. Since the pioneering work of Nowak and May [6], the network reciprocity as one of the significant cooperation mechanisms has been extensively investigated [7]. Szabó and Hauert have studied the voluntary participation in PGG on a square lattice and found that the presence of loners leads to a cyclic dominance of the strategies and promotes substantial levels of cooperation [5]. According to the teaching activity model proposed in Ref. [8], Guan *et al.* have found that the inhomogeneous activity in PGG can remarkably promote cooperation [9]. Szonoki *et al.* investigated the impact of noise for the spatial public goods games on several regular lattices [10]. Recently, Santos *et al.* have explored social diversity by means of heterogeneous scale-free networks and showed that cooperation is promoted by the diversity associated with the number and size of the PGG groups [11]. Wang *et al.* have found the cascade of elimination and emergence of pure cooperation in the PGG [12]. Besides, the collective influence [13], the diversity-optimized behaviors [14], the degree correlation [15], the degree-based and reputation-based partner selections [16] in the PGG are studied on scale-free networks.

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The past ten years have witnessed the rapid development of complex networks theory, spurred by the observations of several features shared in real-world networks [17]. High clustering structure is one of the most important properties, reflected by dense triangles in networks. This common property implies that it is quite possible to find two neighboring individuals sharing the common neighbor [18]. Previous investigations have showed that the clustering property plays a nontrivial roles in the prisoner's dilemma game on regular lattices [19], random graphs [20], small-world networks [21] and scale-free networks [22,23]. More relevant to the subject of this Brief Report, Assenza *et al.* found that the cooperators are flourishing in highly clustered scale-free networks when the temptation to defect is below a threshold value, after which the cooperative behavior is easy to disappear [23]. However, the prisoner's dilemma game is different from the public good games in the sense that the former describes pairwise interactions but the latter are played by groups of individuals. In this Brief Report, we will study the effects of clustering coefficient on cooperation in scale-free public goods games and disclose the feedback reciprocity mechanism of triangle in games played by groups of people.

We adopt the algorithm proposed by Holme and Kim (HK) [24] to obtain the scale-free networks with tunable clustering coefficient (CC), which is the extended Barabási-Albert (BA) model [25]. From the inset of Fig. 1 it is observed that as the increase of a parameter p to tune the prob-

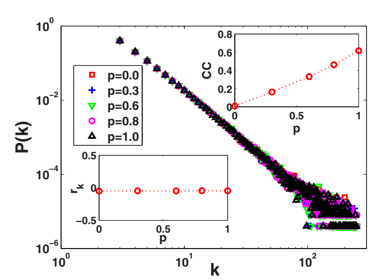

FIG. 1. (Color online) The degree distribution of the HK scale-free networks with different p. The right top inset shows the clustering coefficient CC and the left bottom inset is the degree correlation coefficient r_k versus p on the HK scale-free network with 5000 vertices and averagely six neighbors per vertex. Each data is obtained by averaging over 100 realizations.

ability of forming triangle, both the degree distribution and the degree correlation coefficient [26] keep fixed while the clustering coefficient CC will ascend from 0.0 to 0.6 in the HK scale-free network. Therefore, through the HK model we can study the effect of clustering property on the PGG game with excluding the influences of degree heterogeneity [11] and degree correlation [15].

In the spatial PGG game, each individual i occupies one site on the network, and it participates in k_i+1 games centered on i and its k_i neighbors. According to Ref. [11] that considered the limited resources of individuals and proposed the fixed cost per individual approach, each cooperator (C) with degree k contributes a fixed cost $c=1$ for its neighborhood, which is equally shared among all $k+1$ PGGs. Then, the contribution of a cooperator in one PGG is multiplied by a factor r and divided equally among all participators. Whereas, the defector (D) invests nothing for its neighborhood. Therefore, the payoff of an individual i who participates in the PGG centered on the individual j with degree k_j is: $P_i^j=\frac{r}{k_j+1}\sum_{x\in\Gamma_j}\frac{c}{k_x+1}s_x-\frac{c}{k_i+1}s_i$, where Γ_j means the vertex set of the group containing k_j+1 individuals. We set $s_i=1$ if individual i is cooperator, otherwise $s_i=0$. The total benefit P_i is the accumulated payoffs of PGGs in which i engages. After each generation, all the players update synchronously their strategies by the following rule [27]. Each individual i chooses at random a neighbor j, and in the next generation it will adopt j's current strategy with probability

$$W_{ij}=\frac{1}{1+\exp[(P_i-P_j)/\kappa]}, \quad (1)$$

where κ characterizes the level of rationality of individuals and the cooperative behavior is easy to disappear for larger value of κ. Here, we set $\kappa=0.1$ following the previous investigations [13,14,28]. The larger the payoff difference between j and i, the more probability i tends to learn j's strategy. Note that although the noise is fixed for all vertices, it has different influences for vertices with different degrees. This can be understood in the sense that higher degree vertices due to their more interactions can usually gain higher payoffs P and the noise κ in the denominator of Eq. (1) becomes relevant smaller for higher P. The fixed noise in the Fermi-function thus brings additional diversity effects into the strategy updating. As a result, the strategy of higher degree vertices is more favored by the noise. In one PGG the payoff of cooperators is less than that of the defectors, therefore, the greedy individuals are willing to become defector and the cooperation is very unstable if there are no effective mechanisms to support cooperation. Then we will study the cooperation evolution on the clustered scale-free networks and uncover the reciprocity mechanism of clustering property for the game dynamics.

In this paper, we set initially the individuals select the strategies of cooperation and defection with equal probability 1/2. The frequency of cooperators, f_C, which is a key quantity to characterize the cooperative behavior of system, is defined as the equilibrium frequency of cooperators during the steady state and is obtained by averaging over 10 000 generations after a transient time with 20 000 generations. Each data is the average over 100 realizations, i.e., 10 runs

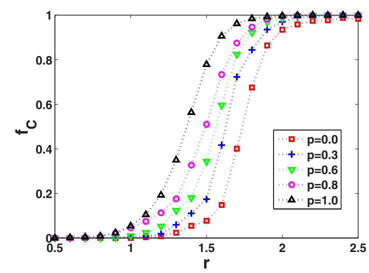

FIG. 2. (Color online) The frequency of cooperators, f_C, as a function of the multiplication factor r when the probability to form triangle, p, increases from 0.0 to 1.0.

for each of 10 different networks with 5000 vertices and average degree $\langle k\rangle=6$. Figure 2 shows f_C as a function of the multiplication factor r for the scale-free networks with different clustering coefficients. It is observed that f_C is an increasing function of r that means the cooperators are easy to survive for large r. More important, it is showed from Fig. 2 that the cooperation can maintain for small r on the scale-free network with high clustering. Below we will explain the mechanism that why the cooperative behavior can boom on the highly clustered scale-free networks.

Previous investigations have successfully disclosed the degree heterogeneity of scale-free network through the two-star subgraph [7,11], and Vukov *et al.* have studied the role of clustering property in the prisoner's dilemma game through two kinds of regular random graph with or without triangles [20]. In this paper, we use two typical starlike subgraph as showed in Fig. 3 to represent the scale-free network without (with) clustering property, where k_i denotes the degree of vertices at the ith layer (denoted as L_i, $i=0,1,2,\ldots$). In the Fig. 3(b) (denoted as starlike-II), the hub at L_0 and two vertices at L_1 compose a triangle. To characterize the degree heterogeneity of the scale-free network, we assume $k_0>k_1$. For simplicity, we set the other vertices have the same degree k_1. Figure 3(a) (denoted as starlike-I) can be obtained by reshuffling an edge linking vertices at L_1 and an edge randomly chosen at faraway layer in starlike-II according to the Mslov-Sneppen algorithm [29]. Since the vertices excluding the hub have the same degree, the reshuffling operation will not change the degree heterogeneity and degree correlation of the original subgraph. And for a large network the randomly selected edge is far from the hub, therefore, we can

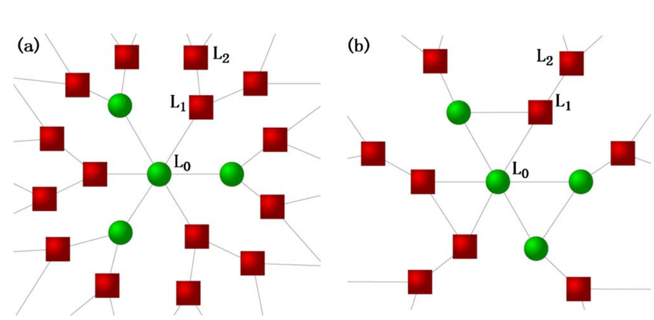

FIG. 3. (Color online) Two typical subgraphs: (a) the starlike-I subgraph without the triangle around hub and (b) the starlike-II subgraph with the triangle around hub. The green (light gray) ball and the red (dark gray) box denote the cooperator and the defector respectively.

obtain a starlike-I subgraph as showed in Fig. 3(a). We focus on the influence of triangle structure to the evolution of cooperation of PGG. Initially we set only the central hub at L_0 and its n_C neighbors at L_1 are cooperators and others are defectors. Then we will investigate the condition of the cooperation diffusion in the two subgraphs through considering the payoff difference between cooperators and defectors. It is noteworthy that although the schematic networks are simple and regular which differ from large scale-free networks, they are helpful to explain the effect of triangular structure on cooperation to some extent by taking into account the presence a hub at the center and a treelike architecture which by excluding loops is available for calculations. As will be demonstrated later, the simple networks can yield insight into understanding the role of clustering structure in scale-free networks, especially for triangles around hubs.

It is showed from Eq. (1) that the value of payoff difference between one vertex j and its neighbor i characterizes the probability that the behavior of the former replaces the latter. Therefore, through investigating the payoff difference between the cooperator and its defective neighbor, we can study how the cooperative behavior diffuses on the network. We first study the payoff difference between cooperator and defector in the starlike-I subgraph showed in Fig. 3(a). Consider the C-hub at L_0 will invest $1/(k_0+1)$ for each PGG centered on itself and its L_1 neighbors, and the n_C cooperative neighbors at L_1 will contribute $1/(k_1+1)$ for each PGG focal on themselves and C-hub, therefore, the payoff $P^C_{L_0}$ of the C-hub is

$$P^C_{L_0} = \frac{rc}{k_0+1}\left(\frac{1}{k_0+1} + \frac{n_C}{k_1+1}\right) + \frac{rc}{k_1+1}\left(\frac{k_0}{k_0+1} + \frac{n_C}{k_1+1}\right) - c. \tag{2}$$

Since the defector at L_1 can obtain benefit from C-hub through the two PGGs centered on itself and C-hub, respectively, we can obtain the payoff difference between the C-hub and its defective neighbor at L_1 as

$$P^C_{L_0} - P^D_{L_1} = \frac{rc}{k_1+1}\frac{k_0-1}{k_0+1} + \frac{rc}{k_1+1}\frac{n_C}{k_1+1} - c = \alpha_1 c, \tag{3}$$

where $\frac{rc}{(k_1+1)(k_0+1)}$ and $\frac{rc}{(k_1+1)^2}$ correspond to the contributions of the C-hub and L_1-cooperators for the PGGs centered on individuals at L_1. Similarly, compared with the defector at L_2, the cooperator at L_1 can obtain additional benefits from the PGGs centered on the C-hub and k_1-2 defective neighbors at L_2. Therefore, the payoff difference between the cooperator at L_1 and its defective neighbor at L_2 is

$$P^C_{L_1} - P^D_{L_2} = \frac{rc}{k_0+1}\left(\frac{1}{k_0+1} + \frac{n_C}{k_1+1}\right) + \frac{rc}{k_2+1}\frac{k_1-2}{k_1+1} - c = \alpha_2 c. \tag{4}$$

Then we turn to investigate the payoff difference between cooperators and its defective neighbors in the starlike-II subgraph showed in Fig. 3(b). Since the existence of triangle, the C-hub can obtain the contribution of a cooperative L_1-neighbor i not only from the PGG centered on i, but also from the common neighbor with i. Therefore, the payoff of the C-hub in the starlike-II subgraph is: $P^C_{L_0}=\frac{rc}{k_0+1}(\frac{1}{k_0+1}+\frac{n_C}{k_1+1})+\frac{rc}{k_1+1}(\frac{k_0}{k_0+1}+\frac{2n_C}{k_1+1})-c$. It should be noted that compared with the Eq. (2), the C-hub in the starlike-II subgraphs can obtain additional $\frac{rcn_C}{(k_1+1)^2}$ payoff since the L_1-cooperators can feed back their investment to C-hub through the triangle. Furthermore, we consider the worst case that the C-hub and its defective L_1-neighbor j share one cooperator i. The defector j can get benefits from the three PGGs centered on the C-hub, i and j, respectively, and thus the payoff difference between the C-hub and the defector j is

$$P^C_{L_0} - P^D_{L_1} = \frac{rc}{k_1+1}\frac{k_0-2}{k_0+1} + \frac{rc}{k_1+1}\frac{2(n_C-1)}{k_1+1} - c = \beta_1 c, \tag{5}$$

Furthermore, we can obtain the payoff difference between the cooperator i and its defective neighbor at L_2 as

$$P^C_{L_1} - P^D_{L_2} = \frac{rc}{k_0+1}\left(\frac{1}{k_0+1} + \frac{n_C}{k_1+1}\right) + \frac{rc}{k_1+1}\left(\frac{1}{k_0+1} + \frac{1}{k_1+1}\right) + \frac{rc}{k_2+1}\frac{k_1-3}{k_1+1} - c = \beta_2 c, \tag{6}$$

where the first, second and third term of Eq. (6) corresponds to the gains of i obtaining from the PGGs centered on the C-hub, the L_1- and L_2-neighbors with defective strategy, respectively.

In order to understand the distinction between two subgraphs in the aspect of the ability of cooperation diffusion from L_0 to L_1, we compare Eq. (5) with Eq. (3) and can find that: $\beta_1-\alpha_1=\frac{r}{k_1+1}(\frac{n_C-2}{k_1+1}-\frac{1}{k_0+1})$, which implies that if $n_C>2$ and $k_0>k_1$, the reciprocity between the C-hub and its L_1-cooperators can be enhanced through the feedback loops of triangle, therefore, $\beta_1>\alpha_1$ and the invasion from C-hub to the L_1-defectors is easier to occur in the starlike-II subgraph than that in the starlike-I subgraph. This forms a positive feedback that if there are more cooperators around the C-hub, it can obtain more payoff and the cooperative behavior is easier to diffuse from L_0 to L_1. Furthermore, comparing Eq. (6) with Eq. (4) and considering $k_1=k_2$, we can obtain the ability distinction of cooperation diffusion from L_1 to L_2 as $\beta_2-\alpha_2=\frac{r}{(k_0+1)(k_1+1)}$. It is observed that since the C-hub can feed back the investment to its cooperative neighbor through the triangle, $\beta_2>\alpha_2$ and the L_1-cooperators is easier to diffuse their behaviors to L_2-neighbors on the starlike-II subgraph than that on the starlike-I subgraph. This also leads to the increase of C-hub's payoff and speeds up the diffusion of cooperation on the network. Therefore, the triangle clustering in the scale-free network can form a kind of feedback reciprocity that promotes the diffusion of cooperation. Although all the analysis is carried out from C-hub initially, the scenario of starting from a D-hub can be interpreted by the current analysis as well. It has been known that the D-hub will trigger a negative feedback due to the diffuse of its defection strategy to leaves. The D-hub will be quite likely to turn to C if it connects to some C-hubs [11]. After the strategy switch of the D-hub, the following evolution can be cast into our analysis with a preliminary C-hub.

Finally, we would like to check the feedback reciprocity mechanism on the clustered scale-free networks. We set initially the highest-degree vertex and some low-degree neighbors to cooperators and others to defectors on the scale-free network. Then we study time series of the frequency of cooperators and investigate how cooperation diffuses on the scale-free networks with different clustering coefficients. This method is also used in Ref. [30] to study the spatial invasion of cooperators who play the prisoner's dilemma and

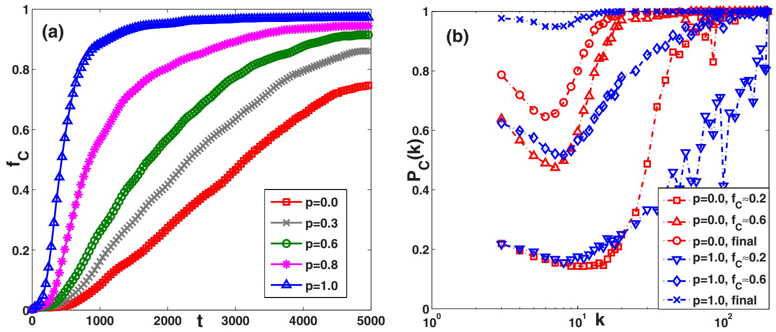

FIG. 4. (Color online) (a) The frequency of cooperators f_C versus the generation t for scale-free networks with different clustering coefficients. Each network contains 5000 vertices and averagely each vertex has 6 neighbors. Initially there are only ten cooperators (the highest-degree vertex and its nine low-degree neighbors) and others are defectors. (b) is the fraction of cooperators per degree, $P_C(k)$, for different instants of the time evolution. The data are obtained by averaging over ten different networks with 50 runs for each network. The multiplication factor $r=1.8$.

snowdrift games on the square lattice. We observe from Fig. 4(a) that the higher clustering property the network has, the easier the cooperation diffuses and the earlier the frequency of cooperators reaches to the equilibrium state. Furthermore, it is showed from Fig. 4(b) that there are more cooperators at the low-degree vertices in the higher clustered scale-free network than that in the lower case for both the final state and the transient states with the similar f_C, which implies the cooperation is easier to diffuse from hub to low-degree vertices in the higher case. Therefore, it needs longer transient time in the lower case until sufficient high-degree vertices become cooperators. These results are consistent with our previous analysis of the feedback reciprocity mechanism of triangle in the subgraphs.

In conclusion, we have studied the influence of clustering coefficient on the evolution of cooperation in the scale-free public goods games. Through comparing the payoff difference between cooperators and defectors in two types of star-like subgraphs, we discover the feedback reciprocity mechanism of clustering property. Since in the spatial PGG the payoff of an individual is not only related with its immediate neighbors, but also depends on its neighbors' neighbors, the triangle loop makes the cooperator obtain the additional investment of another cooperator from their common neighbor, which is different from pairwise interactions in the spatial PDG where an individual only gains payoffs from its immediate neighbors. Relying on the feedback reciprocity of triangle in PGG, the cooperative behavior becomes easier to diffuse on the highly clustered scale-free networks. Our investigation yields some insight into understanding the mechanism of cooperation emergence and designing the proper network topology to boom cooperation.

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