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Self-organized emergence of navigability on small-world networks

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Abstract. We study the origin of navigability in small-world (SW) networks and propose a general scheme for navigating SW networks. We find that navigability can naturally emerge from self-organization in the absence of prior knowledge about the underlying reference frames of networks. Through a process of information exchange and accumulation on networks, a hidden metric space for navigation on networks is constructed. Navigation based on distances between vertices in the hidden metric space can efficiently deliver messages on SW networks, in which long-range connections play an important role. Numerical simulations further demonstrate that a high cluster coefficient and a low diameter are both necessary for navigability. These interesting results provide profound insights into scalable routing on the Internet due to its distributed and localized requirements.

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1. Introduction

Small-world (SW) networks are ubiquitous in nature and society. In the late 1960s, Travers and Milgram [1] discovered the SW phenomenon by studying the delivery of letters among people. In the experiment, each participant could only deliver letters to a single acquaintance who was more capable of delivering letters to the target persons in their opinion. Relying on this greedy routing strategy, or the so-called navigation, at last 29% of the letters reached the target persons and the average length of acquaintance chains of letters that were successfully sent was 6. In 2001, Watts *et al* performed a global and Internet-based social search experiment involving more than 60 000 individuals aiming at reaching one of 18 targets in 13 countries by forwarding email messages to acquaintances. It was found that the successful social search with an average of 4.1 hops was conducted primarily through intermediate to weak strength ties [2]. Experiments performed by Adamic and Adar in 2004 proved that the SW searching strategy, by greedy forwarding on the email network of HP Labs based on physical space or the organizational hierarchy relative to the target, could effectively locate most individuals within four steps [3]. Moreover, a recent work proposed by Liben-Nowell *et al* [4] demonstrated that the greedy routing strategy based on geographic position was efficient in passing messages together with the median path length 4 on large scale-social networks. These striking results suggest that people are connected with much shorter chains than we can imagine, and they can find the short paths based solely on local information, regardless of the network size and the topological distances between people.

The navigability of SW networks has attracted a great deal of interest among scientists. A variety of network models have been proposed to explain the underlying mechanisms that ensure finding the shortest paths based exclusively on local information. In these models, networks were generated based on the underlying reference frames (e.g. grids, hierarchy and hyperbolic spaces) that determined how networks were organized and provided the definitions of distances between vertices. In this regard, adjacent vertices were more likely to be connected by greedy routing that was efficient in passing messages if vertices were aware of the positions of their neighbors and targets. For instance, Kleinberg proved that the navigability of lattices could be improved by adding long-range connections according to distance-dependent probability. The chain length of greedy routing based on geography was bounded by $(\ln N)^2$ if the probability that two vertices link was inversely proportional to the square of their distance [5]. Watts *et al* [6] assumed that individuals belonged to groups embedded in a hierarchy that

defined the ‘social distance’ between individuals, and people tended to connect to those who are socially close to them in the hierarchy. Boguñá *et al* have generated Internet-like scale-free networks with high navigability by embedding vertices into hyperbolic metric space and adding links between nearby vertices according to probability, which decreased with the growth of the distance between vertices [7]–[10].

In fact, the aforementioned works suggest that networks act as an overlay on their underlying reference frames during navigation. Therefore, the navigability of networks is based on the fact that the underlying reference frames are navigable. In these models, efficient navigation needs prior knowledge about the organization of networks. However, several real large-size networks, e.g. email networks and online social service networks, are self-organized, so that it is hard for individuals to be aware of underlying reference frames and to discover their exact positions. Moreover, the impact of network structure on navigability was also investigated. de Moura *et al* [11] studied the ‘look-up’ time of greedy forwarding in the Watts–Strogatz (WS) family of networks based on the distance along the ring between two vertices. They found that the Kleinberg power-law decay of connection distribution was not necessary for fast navigation with local information. Motter *et al* [12] proved that a level of correlation between groups forming a hierarchy of the social structure was important for navigability, leading to a more searchable network, by using scaling analysis and numerical computation.

Here, we aim to address the navigability of SW networks through a different method: establishing a general scheme for efficient navigation by embedding existing networks into hidden metric space, rather than adding links between vertices to build navigable networks. The embedding approach has been considered for predicting network distance on the Internet. This is implemented by embedding networks into a Euclidean space and ensuring that distances between vertices match the shortest path through a proper embedding algorithm, regardless of underlying reference frames [13]–[15]. Other types of network embedding algorithms for the reconstruction of underlying reference frames have also been proposed, e.g. embedding the networks generated by Kleinberg’s model into the Euclidean plane and reconstructing the dimension of the underlying lattice when the network is generated by long-range percolation [16, 17].

Inspired by the fact that the information about acquaintances that is used to evaluate the ‘social distances’ is exchanged through communication in social networks, we embed existing networks into the hidden metric space through self-organization of individuals in networks, regardless of the underlying reference frames. Therefore, the embedding algorithm, in which vertices exchange their position information with their immediate neighbors and decide their positions by themselves, is distributed and localized. The distributed and localized features make the embedding algorithm quite different from previous ones, which required global information, central control or prior knowledge of underlying reference frames. It is demonstrated by numerical simulations that the self-organized algorithm can establish a scheme for efficient navigation, irrespective of the underlying reference frames of networks, and we find that the navigability of networks is influenced by SW properties.

2. Algorithm to establish the navigation scheme

The key to addressing navigability lies in the self-organized embedding algorithm in the absence of prior knowledge about underlying reference frames. In our algorithm, an m -dimension Euclidean space is chosen as the metric space to define distances between vertices. Then we

follow the self-organized process of information exchange and accumulation on social networks, which is described as

$$\mathbf{x}_{i,t} = f(\mathbf{x}_{j,t-1}), \quad j \in \mathbf{N}_i, \quad (1)$$

$$\mathbf{p}_{i,t} = \mathbf{p}_{j,t-1} + \mathbf{x}_{i,t}, \quad (2)$$

where \mathbf{N}_i is the set of immediate neighbors of vertex i . Vectors \mathbf{x} and \mathbf{p} consist of m elements corresponding to the m dimensions of metric space. The vector \mathbf{x} is coupled through the network topology and simultaneously updated according to equation (1), while the position vector \mathbf{p} is the cumulative summation of the historical vector \mathbf{x} . Since information exchange in equation (1) is restricted to between vertices and their direct neighbors, the algorithm is distributed and localized. Meanwhile, distances between vertices will be constant if the vector \mathbf{x} can converge after sufficient evolving steps. Moreover, vertices can be seen as flocking in a metric space, and the vectors \mathbf{x} and \mathbf{p} represent velocity and position, like in the Vicsek model [18]. Velocities of tightly connected vertices synchronize more quickly. Therefore, vertices connected by shorter paths will gather in the metric space, which ensures that the distances between vertices in the metric space are associated with path lengths on networks. Messages can be delivered along short paths by navigation based on distances in the metric space.

Many dynamics can be applied as a realization of equation (1), such as chaotic oscillators coupled by networks that can synchronize depending on suitable coupling strengths. For the purpose of simplicity, we choose the updating rule of vector \mathbf{x} as follows: at every time step, the value of \mathbf{x}_i is the average of its neighbors and the initial $\mathbf{p}_{i,0}$ equals $\mathbf{x}_{i,0}$. Then the algorithm can be written as

$$\mathbf{p}_0 = \mathbf{x}_0, \quad (3)$$

$$\mathbf{x}_{i,t} = \frac{1}{d_i} \sum_j \mathbf{x}_{j,t-1}, \quad j \in \mathbf{N}_i, \quad (4)$$

$$\mathbf{p}_{i,t} = \mathbf{p}_{i,t-1} + \mathbf{x}_{i,t}, \quad (5)$$

where d_i is the degree of vertex i . Equations (4) and (5) can be rewritten in matrix form as the combinations of eigenvectors of the normal matrix \mathbf{N} of the network,

$$\mathbf{P}_0 = \mathbf{X}_0 = \mathbf{V}\mathbf{A}, \quad (6)$$

$$\mathbf{X}_t = \mathbf{N}\mathbf{X}_{t-1} = \mathbf{N}^t \mathbf{X}_0 = \mathbf{V}\mathbf{D}^t \mathbf{A}, \quad (7)$$

$$\mathbf{P}_t = \mathbf{P}_{t-1} + \mathbf{X}_t = \mathbf{V} \left(\mathbf{I} + \sum_{i=1}^t \mathbf{D}^i \right) \mathbf{A}. \quad (8)$$

Each row of matrices \mathbf{X} and \mathbf{P} is the vector of velocities and positions of each vertex. Columns of matrix \mathbf{V} are the eigenvectors of \mathbf{N} . The matrix \mathbf{A} consists of a linear combination of coefficients when eigenvectors of \mathbf{N} are chosen as basis vectors. The matrix \mathbf{D} is a diagonal matrix with eigenvalues of the normal matrix on the main diagonal. Because eigenvalues of \mathbf{N} are in the interval $[-1, 1]$, for long enough evolving time t , we obtain the final position matrix $\tilde{\mathbf{P}}$ as

$$\tilde{\mathbf{P}} = \mathbf{V}\mathbf{E}\mathbf{A}. \quad (9)$$

The matrix \mathbf{E} is a diagonal matrix whose i th diagonal element is $1/(1 - \lambda_i)$, where λ_i is the eigenvalue of the normal matrix. It can be seen that eigenvectors corresponding to large eigenvalues play more important roles in the position matrix as a result of the factor $1/(1 - \lambda_i)$.

Since the positions of vertices in the metric space are linear combinations of eigenvectors of the normal matrix, this demonstrates that the embedding can represent network topology, which is reflected by the fact that adjacent vertices in the metric space are connected by shorter paths on the network. The distance between vertices i and j after sufficient evolving time is

$$\begin{aligned} d_{i,j}^2 &= \sum_{l=1}^m \left[\sum_{k=1}^n \frac{a_{k,l}}{1 - \lambda_k} (v_{i,k} - v_{j,k}) \right]^2 \\ &= \sum_{l=1}^m \sum_{k=1}^n \left[\frac{a_{k,l}}{1 - \lambda_k} \right]^2 (v_{i,k} - v_{j,k})^2 + 2 \sum_{l=1}^m \sum_{p=1}^n \sum_{q=p+1}^n \frac{a_{p,l} a_{q,l}}{(1 - \lambda_p)(1 - \lambda_q)} \\ &\quad \times (v_{i,p} - v_{j,p})(v_{i,q} - v_{j,q}), \end{aligned} \quad (10)$$

where $a_{i,j}$ and $v_{i,j}$ are the elements of matrices \mathbf{A} and \mathbf{V} , respectively. If elements of \mathbf{X}_0 are uniformly distributed in the interval $[-1, 1]$, the elements of the matrix \mathbf{A} have the following properties: $\langle a_{i,j} \rangle = 0$, $\langle a_{i,j} a_{k,l} \rangle = 0$ and $\langle a_{i,j}^2 \rangle = \langle x^2 \rangle$. In addition, if m is sufficiently large, the distance can be expressed by

$$d_{i,j}^2 = \sum_{k=1}^n \frac{m \langle x^2 \rangle}{(1 - \lambda_k)^2} (v_{i,k} - v_{j,k})^2. \quad (11)$$

Equation (11) shows that the distances between vertices can be seen as those in the situation where the position values of vertices are elements of weighted eigenvectors of the normal matrix. Due to the factor $(1 - \lambda_k)^{-2}$, distances are mostly determined by eigenvectors associated with large eigenvalues. It has been proved that these eigenvectors are the solutions of the following constrained optimization problem [19]. Let the energy of the system $z(x)$ be defined as

$$z(x) = \frac{1}{2} \mathbf{x}' \mathbf{L} \mathbf{x}, \quad (12)$$

where \mathbf{L} is the Laplace matrix of a network and \mathbf{x} are position values assigned to the vertices together with a constraint,

$$\mathbf{x}' \mathbf{K} \mathbf{x} = 1, \quad (13)$$

where the matrix \mathbf{K} is a diagonal matrix whose i th main diagonal element is the degree of vertex i . Let $\lambda_1 < \lambda_2 < \dots < \lambda_{n-1} < \lambda_n = 1$ be the eigenvalues, and the corresponding eigenvectors under the constraint of equation (13) are v_1, v_2, \dots, v_{n-1} and v_n . The minimum nontrivial value of z is $1 - \lambda_{n-1}$, and the relevant position vector x is v_{n-1} . If the energy reaches the minimum nontrivial value, vertices that are connected by a number of short paths are sufficiently close in the metric space constructed by eigenvectors, which ensures that distances in the metric space correspond to path lengths on networks.

Due to the fact that similar vertices are more likely to be connected, it is natural to evaluate similarities based on the number of paths between vertices and the length of paths in the absence of prior knowledge of underlying reference frames [20]. Through this evaluation, vertices connected by more number of and shorter paths, which will be adjacent in the metric space after self-organized embedding, are deemed to be more similar. Therefore, the results of

the embedding algorithm are consistent with the basic ideas of underlying reference frames: similar vertices are adjacent and more likely to be connected.

3. Experimental results

3.1. Experimental results of small-world (SW) networks generated by the Watts–Strogatz (WS) model

The self-organized embedding algorithm is applied to build a navigation scheme on SW networks generated by the WS model, in which SW properties result from rewiring the edges of the original regular network at probability p [21]. The chosen original regular network has $n = 1000$ vertices, and each vertex links to $k = 10$ nearest others. The diameters and cluster coefficients of networks at different rewiring probabilities are shown in figure 1(a). Experimental results are averaged over 20 network realizations. As shown in figure 1(a), even for the small rewiring probability, the diameters of networks decrease sharply, while the cluster coefficients are nearly the same as those of the original regular network.

At the beginning of the embedding algorithm, every vertex is assigned an initial velocity $\mathbf{x}_{i,0}$, whose values for each dimension are uniformly distributed within $[-0.5, 0.5]$. The dimensions of metric spaces are chosen to be $m = 5, 10$ and 20 to investigate how the metric space influences navigation. The embedding algorithm is terminated when the velocities of vertices reach a certain synchronization level. We defined the synchronization error of \mathbf{x}_i of dimension k at evolving time t as

$$e_t(k) = \frac{1}{n} \sum_{i=1}^n (x_{i,t}(k) - \langle x_{i,t}(k) \rangle)^2. \quad (14)$$

When the synchronization errors of velocities at each dimension are less than a small value, which is chosen as 10^{-4} , the embedding algorithm is terminated.

The greedy routing strategy to simulate navigation on networks can be described as follows. Vertices are aware of positions of their neighbors in the metric space and the positions of targets are transmitted by messages. Messages are passed through current hop to the neighbor closest to targets at each step. To avoid loops, messages are prohibited from neighbors that have been visited. The routing will terminate if the message reaches the target or all the neighbors of the current hop have been visited. We randomly pick 10^4 source and target pairs for every network to be navigated. Note that the navigation is not symmetric, e.g. navigation from vertex i to j is not equivalent to navigation from vertex j to i because the local environments of vertices i and j are different. Efficient navigation is defined by the fact that messages are successfully passed to targets along the shortest paths. Therefore, we examine two metrics to evaluate navigability: the successfully routed rate (the ratio of the number of successfully routed messages to the total number of messages) and the stretch (the average of the ratios of the routing path length and the shortest path length of all messages).

Figures 1(b)–(d) show the successfully routed rates and stretches as a function of rewiring probability p for the hidden metric space of different dimensions. When rewired connections start to emerge, successfully routed rates increase quickly, whereas stretches grow much more slowly until cluster coefficients drop sharply. As a result of different growth rates, both highly successful routed rates and low stretches, which indicate efficient navigation and strong navigability, simultaneously occur when the networks show SW properties and are much more

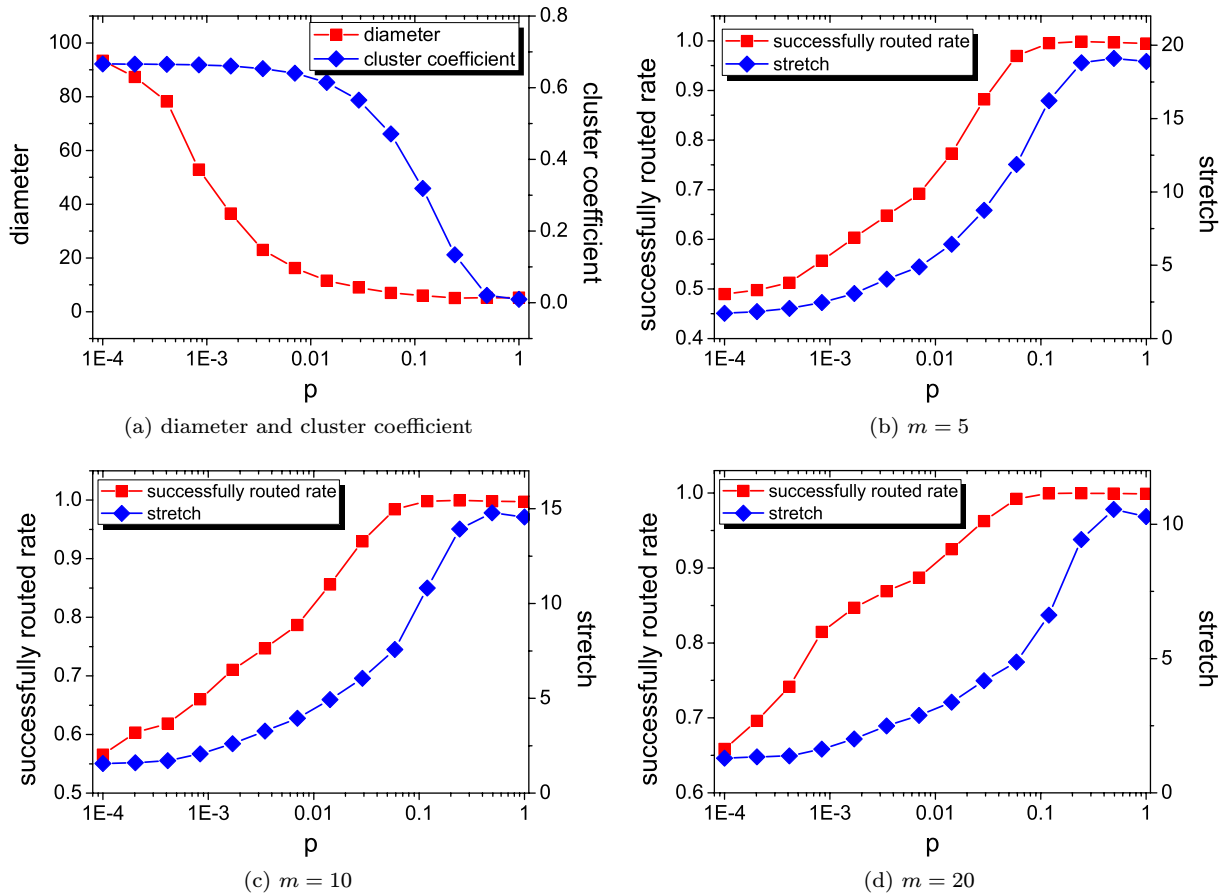


Figure 1. The diameter and cluster coefficient as a function of rewiring probability p (a); the performance of greedy routing for different dimensions of metric space: (b) $m = 5$, (c) $m = 10$ and (d) $m = 20$. Networks are generated by the WS model [21]. Numerical simulations at each p are averaged over 20 realizations of the model. SW networks show strong navigability with high successfully routed rates and low stretches for all dimensions. In particular, the SW properties are necessary for navigability, and the large metric space dimension is useful in improving the navigability.

apparent for the hidden metric space of larger dimensions. In other words, the larger dimension of hidden metric space is useful in improving the performance of navigation, which is reflected by higher successfully routed rates and lower stretches at the same rewiring probability.

Long-range connections, or the so-called weak ties in sociology, play an important role in activities on networks, e.g. information that people receive through weak ties is more useful and successfully routed messages on email networks are conducted primarily through intermediate to weak strength ties [2, 22]. Hence, it is worth studying how long-range connections affect navigation by passing messages to vertices far away from each other on networks. We calculate the distributions of the shortest path length between all the pairs and successfully routed pairs at different rewiring probabilities p when the metric space dimension is 20 (see figure 2). When there are fewer long-range connections, messages passing along a shorter source–target path

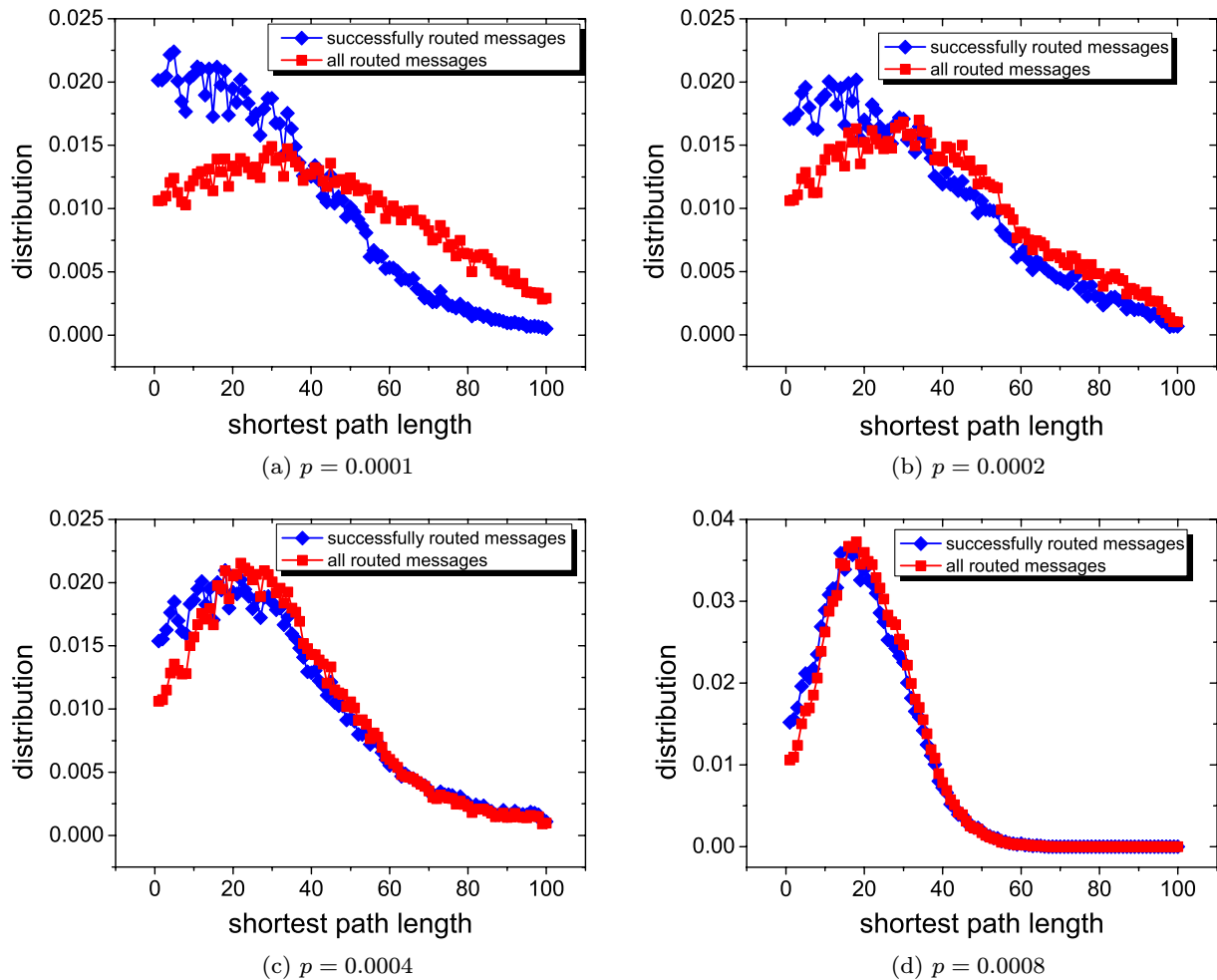


Figure 2. Distributions of shortest path length between the source and the target of all routed messages and successfully routed messages for different rewiring probabilities p : (a) $p = 0.0001$, (b) $p = 0.0002$, (c) $p = 0.0004$ and (d) $p = 0.0008$. As the number of long-range connections increases, messages can escape from the local area of source vertices and travel a long distance. The two distributions have agreed well with each other even when the rewiring probability is still very small. These results reflect the power of weak ties: even if a few long-range connections exist, messages could be passed to the entire network.

take a greater percentage of successfully routed messages than in all messages. The reason is that many messages cannot travel far away from the starting vertices on networks with high cluster coefficients resulting from quickly arriving at those vertices whose neighbors have all been visited. As the number of long-range connections increases, messages can escape from the local area of source vertices and travel a long distance on networks to arrive at targets. Therefore, targets are successfully reached at the same probability for different path lengths from sources. Moreover, the two distributions have agreed well with each other when the rewiring probability is 0.0008, which results in a sufficiently small number of long-range connections compared with

the total number of connections. The weak ties are extremely useful in the sense that even if a few long-range connections exist, messages could be passed to the whole network. This fact also explains why the successfully routed rates immediately increase fast when there are only a few long-range connections.

The navigability of networks in terms of the self-organized embedding algorithm is based on the fact that distances in the metric space are associated with the similarities of vertices extracted from topology. However, we cannot ensure that the distance between every vertex pair represents its similarity in the absence of central control, e.g. adjacent vertices in the metric space may not be tightly connected. Actually, greedy routings performed on networks consist of two parts: the properly directed part has a relevant to navigation, while the remaining part is more tendency to random walks. For numerical simulations on SW networks, when cluster coefficients stay at a high value, there are clusters consisting of tightly connected similar vertices, which satisfies the organizing rules of networks based on underlying reference frames. In this case, network topology can be mapped into a hidden metric space by the self-organized embedding algorithm. Meanwhile, messages cannot travel along paths through random walk on networks with high cluster coefficients because they are easy to reach a vertex, all of whose neighbors have been visited. Successfully passed messages on highly clustered networks are mostly routed by navigation, which leads to low stretches. When cluster coefficients drop quickly, the local clusters vanish by randomly rewired connections and vertices are randomly placed in the hidden metric space, which differentiates the embedding of networks from the network topology. In this regard, random walks can travel a long path to reach targets because vertices have few common neighbors. Therefore, most messages are successfully delivered by random walks, which leads to large stretches and the successfully routed rates are close to 1.

3.2. Experimental results of SW networks with power-law degree distribution

Many real SW networks have the power-law degree distribution $p(k) \sim k^{-\gamma}$, for example the Internet and WWW. They are called scale-free networks, in which there are vertices with much larger degrees than those of randomly connected networks, such as the Erdős–Rényi (ER) model. The largest degree of a scale-free network is proportional to $N^{1/(\gamma-1)}$, where N is the number of vertices in the network. The Barabási–Albert (BA) model has been proposed to explain the emergence of power-law degree distributions based on the ideal of preferential attachment [23]. We also investigate the navigability of scale-free networks generated by the generalized BA model [24, 25]. In this model, a vertex is added in the network with m connections at each step. The probability of attaching to an existing vertex of degree k is proportional to $k + k_0$, where the offset k_0 is a constant. Note that k_0 being larger than $-m$ is to ensure positive probabilities. This model yields a power-law degree distribution with the exponent $\gamma = 3 + k_0/m$. Negative values of k_0 lead to an exponent less than 3, which has been observed in many real complex networks.

The scale-free networks consist of 10^3 vertices together with $m = 3$ and the offset k_0 being turnable to obtain exponent γ from 2 to 4. Results for different γ are averaged over 20 networks. SW properties of scale-free networks are shown in figure 3(a). Scale-free networks with smaller exponents exhibit stronger SW property as the cluster coefficients quickly decrease with the growth of the exponent. We construct a metric space and exactly execute greedy routing similar to SW models. Figure 3(b)–(d) show the performance of navigation for different exponents γ . This demonstrates that when networks exhibit SW properties with small γ , strong navigability emerges. Stretches are also affected by cluster coefficients because the topology of highly

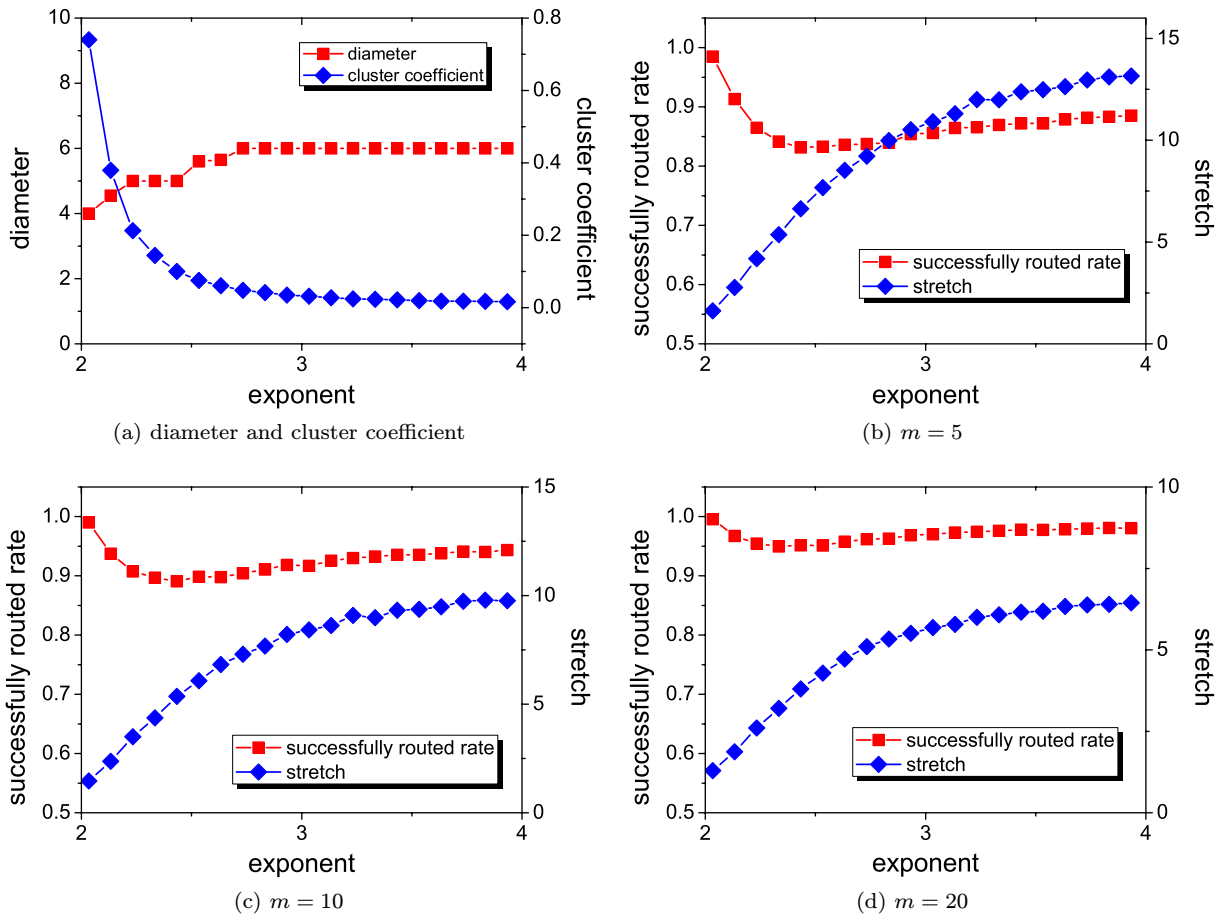


Figure 3. The diameter and cluster coefficient as a function of the exponent γ (a); the performance of greedy routing for different dimensions of the metric space: (b) $m = 5$, (c) $m = 10$ and (d) $m = 20$. Networks are generated by the generalized BA model [24, 25]. Experimental results at each γ are averaged over 20 realizations of the model. Scale-free networks with a small exponent γ show strong navigability represented by high successfully routed rates and low stretches for all dimensions. Like the results of SW models, both a high cluster coefficient and a low diameter are also necessary for navigability, and the large metric space dimension is also helpful in improving navigability.

clustered networks can be more properly mapped into a metric space. In addition, it can be seen that the high degree nodes act as hubs in navigation on scale-free networks [8]. Therefore, as γ increases, successfully routed rates slightly drop because the highest degrees decrease. However, when cluster coefficients continuously decrease, successfully routed rates start to increase because most messages are passed by random walks, which also leads to large stretches.

4. Conclusion

In conclusion, we have substantiated the emergence of navigability on SW networks by mapping the network topology into Euclidean hidden metric spaces through a simple embedding

algorithm based on information exchange and accumulation in the absence of prior knowledge of underlying reference frames of networks. It has been demonstrated that high navigability emerges only if networks exhibit strong SW properties. Despite the lack of prior knowledge about underlying reference frames, the self-organized embedding algorithm can establish a navigable scheme for different kinds of SW networks, which is supported by the results of SW networks generated by the WS model and the BA model.

The self-organized navigation may be a possible approach available for scalable routing on the Internet, which has attracted much interest recently. Many algorithms have been proposed for reducing the storage space of the routing table without a remarkable increase in the routing path lengths, e.g. the compact routing schemes [26]–[28]. The size of routing table could be reduced to poly-logarithmic of the network size in compact routing with a stretch smaller than 3, yet global topology and central control required for building the routing scheme in these algorithm will demand a large amount of communications on networks. In this work, since the constructing hidden metric space and greedy routing are distributed and localized in a self-organized way, communications are restricted between immediately connected vertices. Meanwhile, the sizes of routing tables are the degrees of vertices, and stretches are quite small when the networks show SW properties. Compared with previous works on navigation [5]–[10], this work may provide profound insights into the scalable routing scheme through a self-organized method in the absence of prior knowledge.

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