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The effect of bandwidth in scale-free network traffic

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Abstract – This paper models the effects of bandwidth on the traffic capacity of scale-free networks. We investigate the decrease of the system traffic capacity and the variation of the optimal local routing coefficient α_c, induced by the restriction of bandwidth. For low bandwidth, the same optimal value of α_c emerges for two different cases of node capacity, namely $C = constant$ and $C_i = k_i$, where k_i denotes the degree of the i-th node. By investigating the number of packets at each node in the free-flow state, we provide analytical explanations for the optimal value of α_c. Average packet travelling time, distribution of packet travelling time, and average visits per node divided by the node connectivity are also studied.

Introduction. – Complex networks theory has attracted growing interest among the physics community since the pioneering discovery of the small-world phenomenon [1] and scale-free property [2]. Complex networks can be used to model many natural, social and technical systems in which a lot of entities or people are connected by physical links or abstract relations [3–8]. Due to the importance of large communication networks such as the Internet, WWW, power grid and transportation systems with scale-free properties in modern society, the traffic of information flows has attracted more and more attention. Ensuring free traffic flows on these networks is of great significance and research interest [9–32].

Various models have been proposed recently to mimic the traffic on complex networks by introducing the concepts of packets generating rate and the routing of packets [10–23]. This kind of models defines the capacity of networks by the critical generating rate at which a phase transition from free-flow state to congested state occurs. The free-flow state corresponds to the state in which the numbers of created and delivered packets are balanced, while the jammed state corresponds to the state in which packets accumulate on the network. To control the congestion and improve the efficiency of transportation, many studies have focused on two aspects: modifying underlying network structures or developing better route searching strategies in large networks [33]. Due to the high cost of changing the infrastructure, the latter is comparatively preferable. In this light, various models have been proposed to forward packets using the shortest path [24], the efficient path [25], the nearest-neighbor and next-nearest-neighbor searching strategy [28–30], the local static information [28], or the integration of static and dynamic information [26,27,29]. In view of the difficulty of knowing the topology of an entire network for many large and rapidly growing communication systems, the local routing strategy attracts more attention because the local static topology information can be easily acquired and stored in each router.

However, previous studies usually neglect the bandwidth of the links, *i.e.*, the maximum capacity of each link for delivering packets. Obviously, in real systems, the capability of each link is limited and changes from link to link and in most cases, these restrictions contributes to the triggering of congestion. As the effects of the bandwidth of links have not yet been analyzed, in this paper, we study the traffic dynamics in which the bandwidth is taken into account based on the local routing strategy.

The traffic model. – To generate the underlying traffic network, our simulation starts with the most general Barabási-Albert scale-free network model which is in good accordance with real observations of communication networks [3]. Driven by the "growth" and "preferential attachment" mechanisms, it can generate a power law degree distribution $P(k) \sim k^{-\gamma}$ with $\gamma \sim 3$. In this model, starting from m_0 fully connected nodes, a new node with m links is added to the existing graph at each time step according to the preferential attachment. The probability Π_i for the new node being connected to the existing node i is proportional to the degree k_i of the node, $\Pi_i = \frac{k_i}{\Sigma_j k_j}$, where the sum runs over all existing nodes.

For simplicity, we treat all nodes as both hosts and routers for generating and delivering packets and assume that each node can deliver at most C packets per step towards their destinations. The capacity of each link is restricted by bandwidth (B), *i.e.*, each link can handle at most B packets from each end per time step. Motivated by the previous local routing models [28,29], the system evolves in parallel according to the following rules:

1) Packets adding rule: Packets are added with a given rate R (number of packets per time step) at randomly selected nodes and each packet is given a random destination.

2) Packets delivering rule: Each node performs a local search among its neighbors. If a packet's destination is found in its nearest neighborhood, it will be delivered directly to its target and then removed from the system. Otherwise, it will be delivered to a neighboring node n with preferential probability: $P_n = \frac{k_n^\alpha}{\Sigma_i k_i^\alpha}$, where the sum runs over the neighboring nodes, and α is a tunable parameter. The FIFO (first-in-first-out) queuing discipline is applied at each node.

To characterize the system's overall capacity, we investigate the order parameter: $\eta(R) = \lim_{t\to\infty} \frac{\langle \Delta N_p \rangle}{R \Delta t}$, where $N_p(t)$ is the number of packets on the network at time t, $\Delta N_p = N_p(t+\Delta t) - N_p(t)$, $\langle ... \rangle$ denotes taking average over the time window of width Δt. Obviously, $\eta(R) = 0$ corresponds to the free-flow state where the balance between added and removed packets can be achieved. As R increases to a critical value R_c, η suddenly increases from zero, which indicates the occurrence of phase transition from free flow to congestion in which packets begin to accumulate on the network. Hence, the system's capacity can be measured by the value of R_c.

Simulation results and discussions. – In the special case of $C = 10$ and $B \geqslant 10$, the maximum network capacity is $R_c \approx 40$, which is achieved at the optimal value $\alpha_c = -1.0$ [28]. This can be explained as follows: the average number of packets on nodes does not depend on the degree k at $\alpha_c = -1.0$, hence no congestion occurs earlier on some nodes than on other nodes with different degrees.

Then we study the effect of bandwidth on the network capacity in the case of fixed node capacity $C = 10$. In the study, we use constant bandwidth $B = 5, 3$ or 1 for all the links in the network or variable integer bandwidth B randomly selected from integers in the range from 1 to 5 for each of the links. The constant B case corresponds to a uniform bandwidth system, and the random B case corresponds to a system with different bandwidth for each of the links. Figure 1 compares the network capacity R_c for five different cases. It is noted that at a given fixed α value, the network capacity decreases as B decreases. This is because the reducing of bandwidth retards the free flow of packets from one node to the other and thus the network capacity decreases.

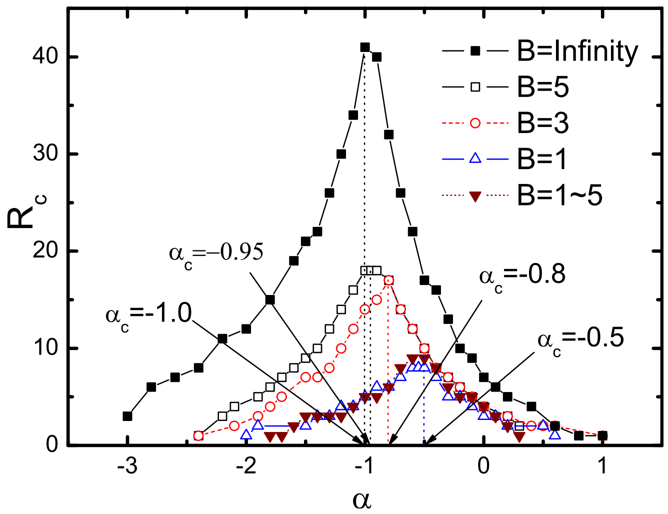

Fig. 1: (Color online). The network capacity R_c against α with network parameter $N = 1000$, $m_0 = m = 3$, constant node delivering ability $C = 10$, and different bandwidth B cases. The data are obtained by averaging R_c over 10 network realizations.

Furthermore, the optimal value of α_c corresponding to the maximum capacity increases from -1.0 to -0.95 for $B = 5$, -0.8 for $B = 3$, and -0.5 for both $B = 1$ and $1 \leqslant B \leqslant 5$. This can be explained as follows. Let $n_i(t)$ denote the number of packets at node i at time t. In the case of homogeneously generated sources and destinations for the packets, the numbers of packets generated and removed at the node i are balanced. Considering the contribution of received and delivered packets of node i to the change of $n_i(t)$, the evolution of $n_i(t)$ in the free-flow state can be written as

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} = -n_{out} + n_{in}, \tag{1}$$

where n_{out} denotes the number of packets delivered from node i to its neighboring nodes, and n_{in} denotes the number of received packets. From eq. (1), in the case of $B \geqslant C$, Wang *et al.* show that $n(k) \sim k^{1+\alpha}$ [28]. Therefore, when $\alpha = -1.0$, the average number of packets on each node is independent of the degree k and thus there will not be some nodes that are more easy to jam, so that the maximum network capacity is achieved. However, $\alpha > -1.0$ means that there are more packets on the hub nodes (with greater degree k). Considering the restriction of $B < C$, since the hub nodes have more links and thus have more total bandwidth, $\alpha > -1.0$ is better in order to fully use the bandwidth of the hub nodes and thus enhance the system's capacity.

To better understand why $\alpha > -1.0$ is the optimal choice, we investigate the number of received packets of

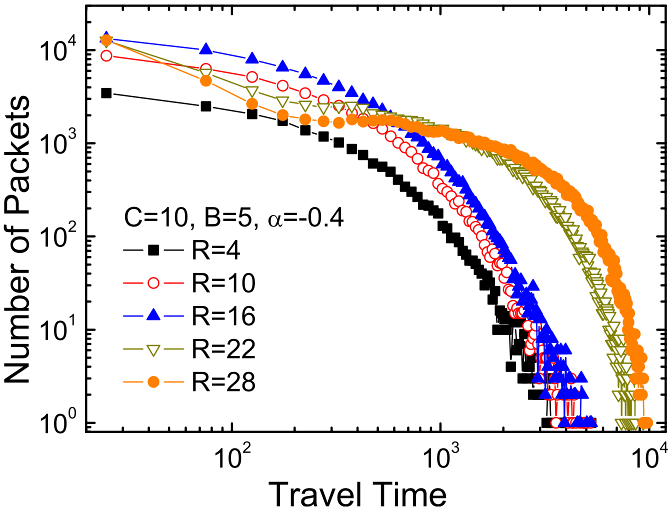

Fig. 2: (Color online). The distribution of packets traveling time with $C=10$. The data are averaged over a period of 5000 time step.

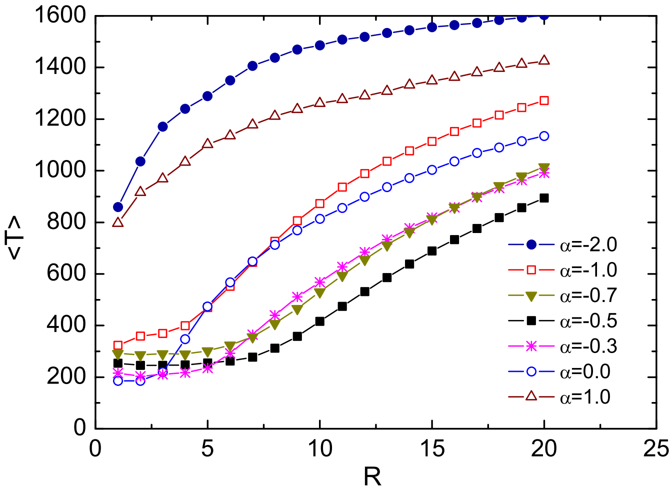

Fig. 3: (Color online). The variation of packets average traveling time $\langle T\rangle$ *vs.* R with different α value. Other network parameters are $N=1000$, $m_0=m=3$, $C=10$ and $B=1$.

node i:

$$n_{in}(i)=\sum_{j=1}^{N}A_{ij}n_jP_i=\sum_{j=1}^{N}A_{ij}n_j\frac{k_i^{\alpha}}{\sum_{l=1}^{N}A_{jl}k_l^{\alpha}}, \quad (2)$$

where the sums run over all the nodes of the network and A_{ij} is the element of the adjacency matrix. Considering that the assortative mixing of BA network is zero, *i.e.*, the average neighbors' degree of each node is the same, we can get $\sum_{l=1}^{N}A_{jl}k_l^{\alpha}=\sum_{l=1}^{N}A_{jl}W=k_jW$, where W is a constant. From eq. (1), one can easily conclude that $n_{out}\geqslant n_{in}$ should be satisfied in order to maintain the free-flow state. For high-degree nodes, n_{out} is mainly constrained by two limits: $n_{out}\approx Bk_i$ and $n_{out}\approx C$. Considering $n_{out}\approx Bk_i$, we can get

$$Bk_i\geqslant\sum_{j=1}^{N}A_{ij}n_j\frac{k_i^{\alpha}}{k_jW}. \quad (3)$$

Since C is a constant, higher-degree nodes are more easily congested than low-degree nodes. We consider the case that i is a high-degree node, for BA scale-free networks, most of the neighbors of i are low-degree nodes. For small B, n_{out} of low-degree nodes are mostly restricted by the bandwidth. Hence, we assume a linear relationship for low-degree nodes: $n_j=Bk_j$. Substituting it into eq. (3), we get

$$\alpha\leqslant\frac{\log W}{\log k_i}. \quad (4)$$

In the limit of very large network, $N\to\infty$, $k_i\to\infty$ and thus the right-hand side of eq. (4) approaches zero, so that the optimal α should be smaller than zero.

Considering $n_{out}\approx C$, we can get

$$C\geqslant\sum_{j=1}^{N}A_{ij}n_j\frac{k_i^{\alpha}}{k_jW}. \quad (5)$$

Substituting $n_j=Bk_j$ to eq. (5), we obtain

$$\alpha\leqslant\frac{\log\frac{CW}{B}}{\log k_i}-1. \quad (6)$$

For low-degree nodes, n_{out} is mainly constrained by Bk_i, *i.e.*, $n_{out}\approx Bk_i$. And for BA scale-free networks, most of the neighbors of low-degree nodes are high-degree nodes, for which we can take $n_j\approx C$. Thus we can get

$$Bk_i\geqslant\sum_{j=1}^{N}A_{ij}C\frac{k_i^{\alpha}}{k_jW}. \quad (7)$$

So we obtain

$$\alpha\leqslant\frac{\log\frac{BW}{W'C}}{\log k_i}, \quad (8)$$

where the constant $W'=\sum_{j=1}^{N}A_{ij}\frac{1}{k_j}$. To continue, we use eq. (6), which is the minimum of eq. (4), eq. (6) and eq. (8). One can see from eq. (6) that the optimal α is achieved when the left side is equal to the right side, and thus α_c is between -1.0 and 0.0. When B decreases from infinity to 1, α_c will be more close to zero. For our simulation parameters, we get $W\approx0.4$ when $\alpha=-0.5$ and $k_{max}\approx100$, and thus $\alpha_c\leqslant-0.7$ for the case of $B=1$. It is quite close to our simulation result.

Then we simulate the packets' traveling time which is also important for measuring the system's efficiency. In fig. 2, we show the distribution of packet traveling time. One can see that when R is small, the distribution has a tail quite close to a power law distribution. But with the increase in R, the distribution deviates away from power law, and more packets spend more time in the system, which implies that the system changes from free-flow to jam. Figure 3 shows the average traveling time $\langle T\rangle$ against R for different α. One can see for $-1.0\leqslant\alpha\leqslant0.0$, $\langle T\rangle$ remains at a relatively small value when $R\leqslant R_c(\alpha)$.

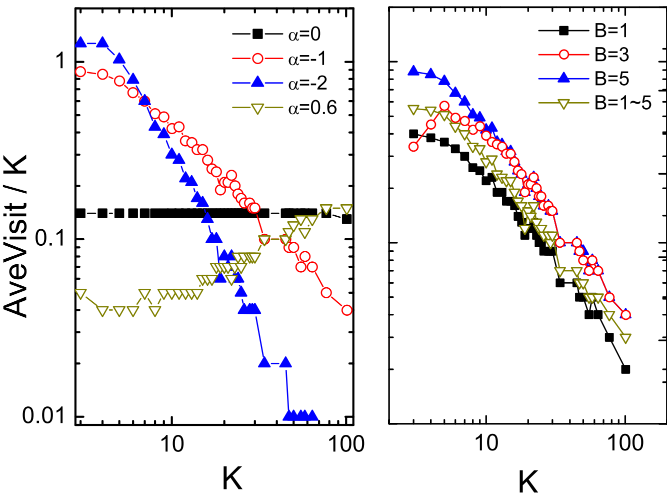

Fig. 4: (Color online). Average visits per node divided by the node connectivity. The two panels show the variation with constant $B=5$, $R=10$ (left), and with constant $\alpha=-1.0$, $R=10$ (right).

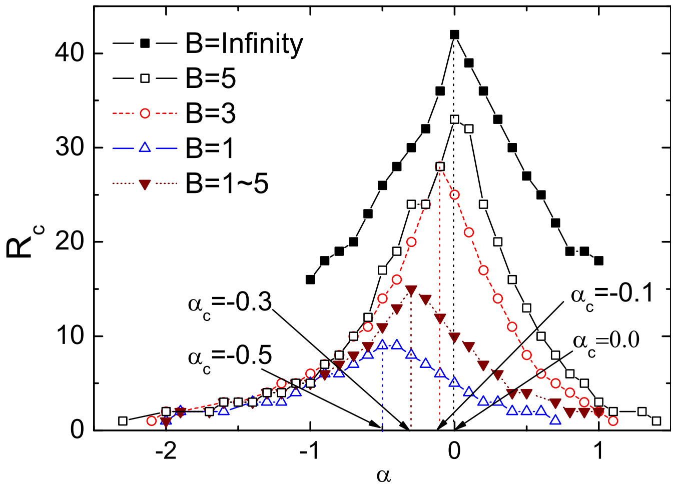

Fig. 5: (Color online). The network capacity R_c against α in the case of $C=k$ with different bandwidth B. The network parameter is $N=1000$, $m_0=m=3$.

When R increases to a value beyond R_c, $\langle T\rangle$ will increase very rapidly, implying that the system is jammed. When $\alpha=-0.5$, the optimal average traveling time is obtained.

Finally, we investigate the average visits per node divided by the node connectivity (denoted as ω) which can be useful in order to analyze the traffic burden distribution among the nodes. Figure 4 shows ω *vs.* k for different α (left panel) and B (right panel). When $\alpha=0$, ω remains at the same value for different k values. This can be explained as follows. In the case of $\alpha=0.0$, packets perform random-like walks on the network. A well-known result in the random walk process valid for this case is that the time the packet spends at a given node is proportional to the degree of such node in the limit of long times [8]. One can easily conclude that, in the traffic system with many packets, the visits per node averaged over a period of time is proportional to the degree of that node. Thus we can get that ω remains constant for different k values. When $\alpha<0$, ω self-organizes to a power law, which implies that the traffic burden of high-degree nodes are alleviated. And this tendency remains the same for different B values. When $\alpha>0$, ω is an increasing function with respect to k, which may lead to the collapsion of hub-nodes.

The $C\sim k$ case. – In the second part, we investigate the effect of bandwidth on the network capacity in the case that C is not a constant but proportional to the degree of each node $C=k$. This may be used to describe the fact that if a router is very important and bears heavy traffic, its delivering ability may be enhanced to avoid congestion.

In the special case of $B\geqslant k_{max}$ corresponding to the maximum degree of the nodes in the network, the main difference from the case of $B\geqslant C=10$ is that the optimal value of local routing parameter α_c changes to 0.0 while the maximum network capacity remains at $R_c\approx 40$ [28]. This is consistent with the previous analysis about the random walk on the scale-free network. One can conclude that in random walk processes, the average number of packets on a given node is proportional to its degree, *i.e.*, $n_i\sim k_i$. At the same time, the node delivering ability C is proportional to its degree, *i.e.*, $C_i\sim k_i$, so that the load and delivering ability of each node are balanced, which leads to a fact that no congestion occurs earlier on some nodes than on others with different degrees. Considering that in the traffic model an occurrence of congestion at any node will diffuse to the entire network, a network with no easily congested nodes has the maximum network capacity, so that routing packets with $\alpha=0.0$ can induce the maximum capacity of the system.

Figure 5 depicts the network capacity R_c against α for different values of B in the case of $C=k$. One can see that the network capacity decreases as B decreases, and the optimal value of α_c also decreases from $\alpha_c=0.0$ for $B=5$ to $\alpha_c=-0.1$ for $B=3$, $\alpha_c=-0.3$ for $1\leqslant B\leqslant 5$, and $\alpha_c=-0.5$ for $B=1$. The reason of the capacity drop is the same as in the case of $C=10$, *i.e.*, the reducing of bandwidth of the link retards the packet delivery process and thus affects the network's overall capacity. The decrease of α_c is different from the case of $C=10$ and can be explained as follows. As mentioned before, $\alpha_c=0.0$ corresponds to $n_i(k)\sim k_i$ and $\alpha_c<0.0$ means redistributing traffic load in hub nodes to other non-central nodes. Considering the free flow condition of $n_{out}\geqslant n_{in}$ with the limitation of $n_{out}\approx C=k$, following a similar analysis, one can get

$$\alpha\leqslant\frac{\log\frac{W}{B}}{\log k_i}. \qquad (9)$$

Or if considering $n_{out}\approx Bk_i$, one can get

$$\alpha\leqslant\frac{\log W}{\log k_i}. \qquad (10)$$

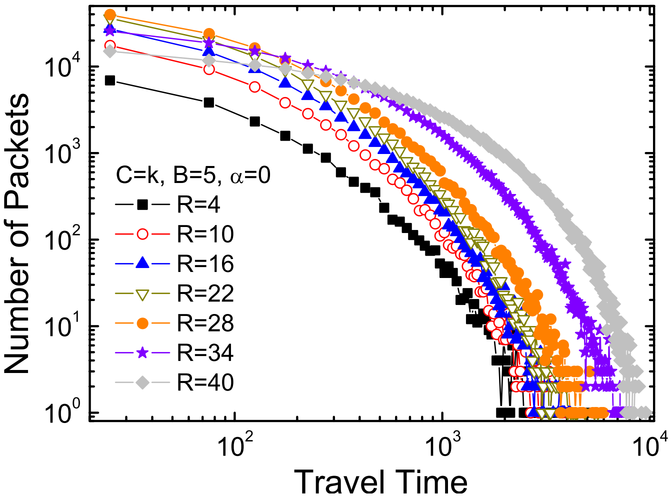

Fig. 6: (Color online). The distribution of packets traveling time with $C = k$.

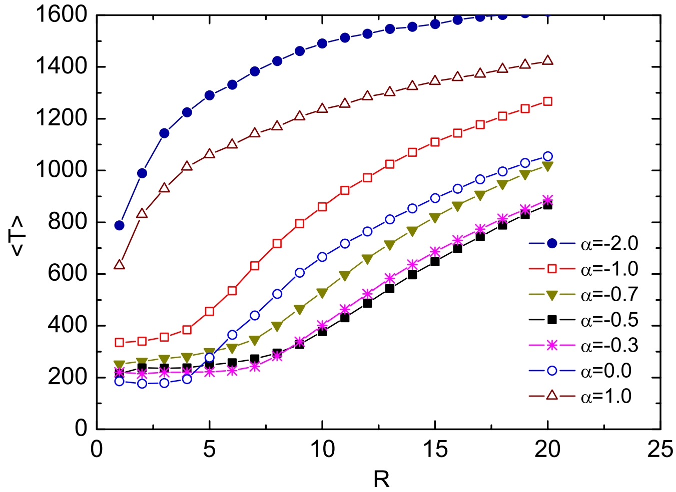

Fig. 7: (Color online). The variation of packets average traveling time $\langle T \rangle$ *vs.* R with different value of α fixed. Other network parameters are $N = 1000$, $m_0 = m = 3$, $C_i = k_i$ and $B = 1$.

In both cases, one can conclude that the optimal α should be close to zero. But when $\alpha = 0.0$, $n_i \sim k_i$ and the nodes perform random selection among all its links to send packets. In the long run of $t \to \infty$, one can find that the number of packets forwarding towards each link in each time step should follow a Poisson distribution with mean value $\lambda = 1$. Thus the hub nodes are more easily jammed when $\alpha = 0.0$. Though the ideal condition is sending one packet per link in each time step, the bandwidth of the link should be more than 1 to maintain free flow, *i.e.*, $B = 1 + \delta$, where δ represents spanning of the Poisson distribution. Therefore, when B is smaller than $1 + \delta$, the optimal α_c should be smaller than zero to redistribute traffic load to other non-central nodes in order to avoid congestion in hub nodes. In fig. 5, one can see that when $B = 5$, α_c remains at zero, whereas when B decreases to less than 5, α_c will decrease from zero.

This is in agreement with Yan *et al.* [25] and Wang *et al.* [29] that redistributing traffic load to the non-central nodes can enhance the system's overall capacity. For α_c smaller than zero, the large degree nodes are fully used, and packets can bypass these nodes when they afford heavy traffic burden. Another interesting phenomenon is that the same $\alpha_c = -0.5$ and $R_c \approx 8$ are obtained for both $C = 10$ and $C = k$ in the case of $B = 1$. This simply shows that the system's capacity is mainly controlled by the bandwidth of the links when the bandwidth B is very low. Thus we cannot improve traffic capacity merely by enhancing routers' ability, because the congestion would be triggered mainly by links.

In fig. 6, we show the distribution of packet travelling time for different α values. One can see that with the increase in R, more packets spend more time in the system. Figure 7 shows the average travel time $\langle T \rangle$ against R for different α when $C_i = k_i$. The results are also in agreement with the above analysis that $\alpha_c = -0.5$ can lead to better efficiency of the network.

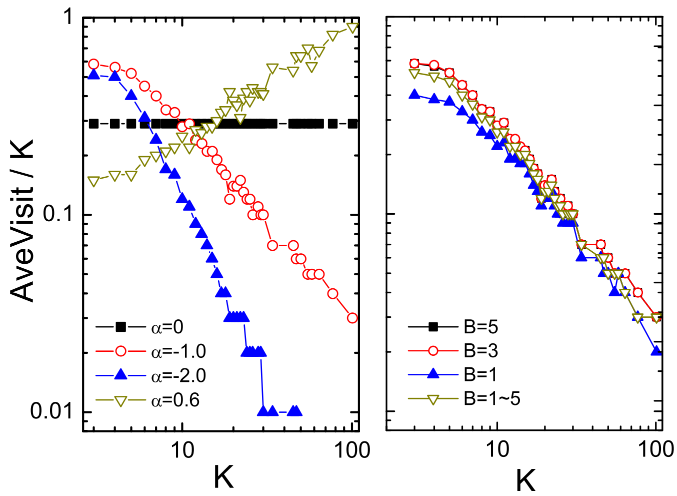

Fig. 8: (Color online). Average visits per node divided by the node connectivity with $C = k$. The left panel shows the quantity with varying α and constant $B = 5$, $R = 10$. The right panel shows the quantity with varying B and constant $\alpha = -1.0$, $R = 10$.

Figure 8 shows the variation of average visits per node divided by the degree of the node (ω *vs.* k) for different α (left) and B (right). The results are similar to those for the case of $C = 10$.

Summary and discussion. – In conclusion, we investigate the effects of the bandwidth of links on the traffic capability in scale-free network base on the local routing strategy. The simulation yields some results different from previous studies. In general, the capacity decreases when the bandwidth of links is considered, whether the node capacity is set as a constant or proportional to the degree of the nodes. Moreover, the optimal value of local routing parameter α_c also depends on the bandwidth of the links, and we found that the node capacity cannot enhance the system efficiency when B is very low. We give analytical explanations for the above phenomena which is in agreement with the simulation results. The distribution of packet traveling time, the average packet traveling time, and the distribution of traffic load among nodes are also investigated.

Our study shows that when considering the effect of bandwidth, the traffic dynamics on scale-free network has many new characteristics. It is also different from traffic on well-organized lattice, on regular or random networks [14,15].

* * *

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