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Traffic dynamics based on an efficient routing strategy on scale free networks

C.-Y. Yin[1], B.-H. Wang[1a], W.-X. Wang[1], G. Yan[2], and H.-J. Yang[1]

[1] Department of Modern Physics, University of Science and Technology of China, Hefei Anhui 230026, P.R. China
[2] Department of Electronic Science and Technology, University of Science and Technology of China, Hefei Anhui 230026, P.R. China

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Abstract. In this paper, we propose a new routing strategy on the basis of the so-called next-nearest-neighbor search strategy by introducing a preferential delivering exponent α. It is found that by tuning the parameter α, the scale-free network capacity measured by the order parameter is considerably enhanced compared to the normal next-nearest-neighbor strategy. Traffic dynamics both near and far away from the critical generating rate R_c are discussed, and it is found that the behavior of $1/f$-like noise of the load time series not only depends on the generating rate R but also on the parameter α. We also investigate R_c as functions of C (capacity of nodes), m (connectivity density) and N (network size). Due to the low cost of acquiring next-nearest-neighbor information and the strongly improved network capacity, our strategy may be useful for the protocol designing of modern communication networks.

PACS. 89.75.Hc Networks and genealogical trees – 89.20.Hh World Wide Web, Internet – 05.10.-a Computational methods in statistical physics and nonlinear dynamics – 89.75.-k Complex systems

1 Introduction

A wide range of systems in nature and society can be described as complex networks. Since the discovery of some interesting common features of many real networks such as small-world phenomena by Watts and Strogatz [1] and Scale-free phenomena by Barabási and Albert [2], the investigation of complex networks, such as the evolution mechanism of network's structure, has attracted more and more attention of the physics community [3–8]. The ultimate goal of any study of network structures is to understand and explain the dynamic processes taking place upon the underlying structures [5]. Thus many kinds of dynamic processes have been investigated in previous works [9–15]. Traffic is one of the important dynamic processes involved in complex networks including the Internet and the WWW. Traffic on regular and random graphs has been extensively explored [16,17]. Due to observations in modern communication networks which show power-law behavior of degree distribution [18,19], it is necessary to study traffic processes on scale-free networks aiming to alleviate the congestion and to enhance the utilization efficiency of the networks.

Random walk and diffusion processes on complex networks [20,21] have been extensively investigated due to their basic dynamic properties and broad application. Their simplicity allows deep theoretical analysis of them. However, these processes are so simple that they could not reflect real traffic systems completely. Recently, some models have been presented in order to describe real traffic systems by introducing the concepts of packet generating rate R, as well as randomly selected sources and destinations of packets [22–25]. For studying the most important phenomenon of phase transition from free flow to congestion, an order parameter is introduced [16]. The critical generating rate R_c at which the phase transition occurs measures the capacity of the system. In the free state, the numbers of created and delivered packets are balanced, resulting in a steady state. However in the congested state, the number of accumulated packets increases with time due to the limited delivering capacity or the finite queue length of each node. Some models utilize the shortest path routing strategy, according to which each packet moves along the shortest path towards its destination [23]. However, for very large communication networks, the cost of acquiring the whole network topology makes it impossible. Some previous works introduced an interesting strategy called next-nearest-neighbor (NNN) searching strategy [22,26]. This strategy can not only significantly enhance the network capacity compared to random walk strategy but also does not require complete graph information. Hence, this strategy is considered to be appropriate for large networks, but it is not the whole

[a] e-mail: bhwang@ustc.edu.cn

story. On the basis of NNN searching strategy, we propose a new routing strategy, namely, preferential next-nearest-neighbor (PNNN) searching strategy which can further alleviate the traffic congestion and greatly improve the packet handling capacity of the network compared to NNN strategy. In PNNN strategy, a parameter α is introduced. The probability that a node i receives packets from its neighbors is proportional to k_i^α in each time step (see Eq. (1)).

In this paper, we treat all the nodes both as hosts and routers. The phase transition from free flow to the jammed state when choosing different α is reported. We find that the network capacity is considerably improved by decreasing α and tends to be stable when α is lower than a specific value. Moreover, by considering the average packet travel time with the maximal network capacity, the optimal parameter value $\alpha = -2$ is obtained. Near the phase transition point, a large fluctuation of traffic load is observed, and the extent of the fluctuation with PNNN strategy is larger than that with the normal NNN strategy in the cases of large α. Another meaningful phenomenon is the exhibition of $1/f$-like noise of power spectrum of traffic load series, which indicates long range correlation [22,24]. If the system shows the behavior of $1/f^2$-like noise, it reflects the zero correlation of the series. The $1/f$ noise is supported by real traffic systems such as vehicular flow in highway networks and data packet flow in computer networks. Simulation results show that when R is far away from the critical packet generating rate R_c, $1/f$ noise emerges. The exponent ϕ of the power spectrum $S(f) \sim f^{-\phi}$ not only depends on R but also correlates with parameter α. The connectivity (link) density of the Barabási-Albert model (BA) can be adjusted by a parameter m [2], thus we also investigate R_c as a function of m. The position of phase transition point influenced by node delivering ability C and network size N is also studied in detail.

The paper is organized as follows. In the next section, the traffic model is described in more detail. In Section 3 numerical simulations are demonstrated in both free flow and the congested state and traffic behavior is discussed extensively as well. In Section 4 this work is summarized.

2 The model

As we have mentioned, the classic ER random graph can not reflect the topology of many real-world networks such as the Internet and the WWW which perhaps possess the self-organized mechanism. Therefore, a famous scale-free network model called the BA model is addressed to mimic the structure of real observations. With the introduced preferential attachment mechanism, the model can generate power-law degree distribution which is in good agreement with the empirical evidence. In this paper, we use the BA model to establish the underlying structure on which the traffic processes take place. Our traffic model is described as follows: at each time step, there are R packets generated in the system with randomly chosen sources and destinations, and all the nodes can post at most C packets towards their destinations. To forward data packets, each node (node l) performs a search on the area of its next-nearest-neighbor. If the target node of a packet is found within the searched area of node l, the packet is forwarded to a neighbor of l which is connected to the target node. Otherwise, if no target exists in the searched area, the packet will be delivered to a node i, one of the neighbors of l, according to the preferential posting probability of that neighbor:

$$\Pi_i = \frac{k_i^\alpha}{\sum_j k_j^\alpha}, \tag{1}$$

where the sum runs over the neighbors of the node l and α is an adjustable parameter. Once the packets arrive at their destinations, they will be removed from the system. Here, the buffer (queue) size of each node is assumed to be unlimited, but the handling and delivering capacity of all the nodes is set to be the same finite constant C. The first-in-first-out (FIFO) rule [16,17,22–25] is applied at each queue of the nodes in our model. Another important introduced rule called path iteration avoidance (PIA) is that a link between a pair of nodes can not be visited more than twice by the same packet. The effect of PIA on the traffic dynamics is discussed in detail in the next section.

The total number of packets existing in the networks N_p is called "load", and the load as a function of time t forms the load series. Two other important quantities for measuring the efficiency of the network is the individual packet travel time τ and the average packet travel time $\langle T \rangle$.

3 Numerical simulation results

The phase transition of traffic systems has been observed both in real-world systems and some traffic models. A natural quantity called the order parameter to characterize the phase transition point more accurately has been introduced [16]. This order parameter is described as follows:

$$\eta(R) = \lim_{t\to\infty} \frac{C}{R}\frac{\langle \Delta N_p \rangle}{\Delta t}, \tag{2}$$

where $\Delta N_p = N(t + \Delta t) - N(t)$ where $\langle \cdots \rangle$ indicates average over time windows of width Δt, and $N_p(t)$ represents the load of the networks at time t. When R is less than a critical value R_c, $\langle \Delta N \rangle = 0$ and $\eta = 0$, which indicates that the system is in the free flow state without the occurrence of traffic congestion. However in the condition of $R > R_c$, $\eta \to r$ (r is a real number more than zero) which results in the global jam of the system. After a long period of time, the system will ultimately collapse. Therefore, the network capacity can be represented by the critical value R_c. Figure 1a reports the order parameter η versus generating rate R for different parameter α. It is easily found that for all different α, η is approximately zero when R is small; it suddenly increases when R is larger than the critical value. Figure 1a also obviously shows that the capacity of the network depends on α. One of the main

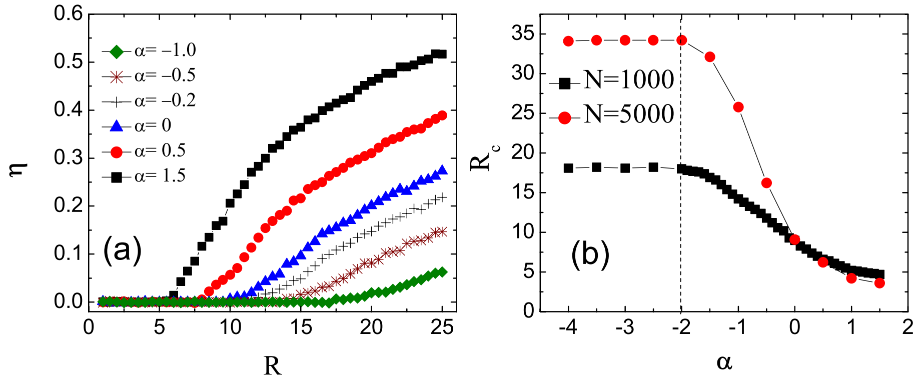

Fig. 1. (a) The order parameter η versus R for a BA network with different parameter α. (b) The critical generating rate R_c as a function of α for network size $N = 1000$ and $N = 5000$, respectively. The other parameters is $C = 1$.

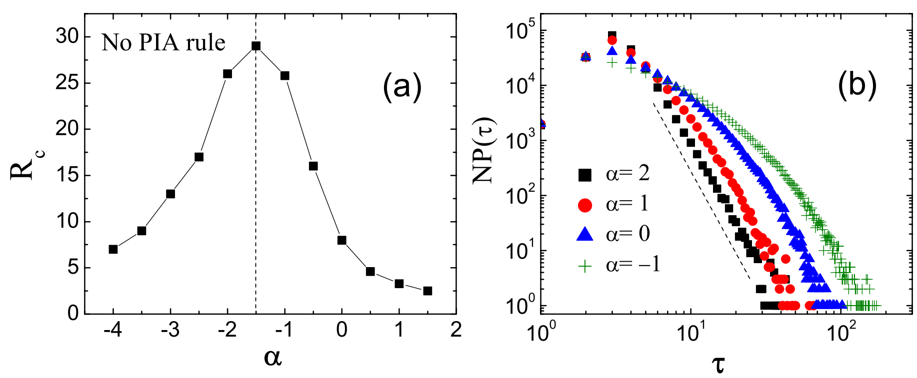

Fig. 2. (a) R_c versus α without the PIA rule for $N = 5000$ and $C = 1$. (b) The packet travel time distribution $NP(\tau)$ versus τ for different α all in the steady state, the simulation lasts for 8000 time steps with $N = 1000$, $C = 1$.

goals of our work is to seek out the optimal value of α corresponding to the maximal network capacity. For this reason, the behavior of R_c versus α for different network size N is explored as shown in Figure 1b. Instead of obtaining an optimal value of R_c, the capacity of the network reaches an upper limit when α decreases to a specific value approximately -2. The simulation results also show that the specific value does not depend on N. In our opinion, the capacity of the network is mainly determined by a few easily congested hub nodes. The effect of reducing α is to allow data packets to bypass those large degree nodes and alleviate the traffic congestion of them. From Figure 1b, we find that compared to the normal NNN strategy corresponding to $\alpha = 0$ of our strategy, the capacity of the network is strongly improved with α decreased to a specific value $\alpha = -2$. However, with further reducing α, the network capacity tends to a steady value. In order to explain this phenomenon, we investigate the behavior of R_c as a function of α without PIA. In Figure 2a, it is found that for $N = 5000$, the system possesses an exclusive maximal capacity point corresponding to $\alpha = -1.5$. Thus the phenomenon that the network capacity reaches an upper limit is ascribed to the PIA. Moreover, compared to Figure 1b, one can find that PIA enhances the maximal capacity approximately from 29 to 34. Because unnecessarily repeated visits of the packets along the same link are prevented by PIA, the packet's transmission speed is improved. Therefore the network capacity is enhanced by introducing PIA.

Packet travel time is also an important factor for measuring the efficiency of the network. The travel time τ of a packet is defined as the time spent by the packet between its origin and destination. Observations of real computer networks reveal that the packet travel time has a power-law distribution, which means that most packets reach their destination in a short time while a few packets spend a very long time within the network. Figure 2b reports the packet travel time distribution $NP(\tau)$ versus τ for different α in the free flow state. In the steady state, almost no congestion on nodes occurs and the time of packets waiting in the queue is negligible, therefore, the travel time is approximately equal to the actual travelling path length of the packets. The simulation results show that when α is large, power-law behavior emerges, which is in accordance with the empirical evidence. Otherwise, the distribution exhibits exponential distribution. Large α in our model denotes that packets tend to move to the nodes with large connectivity in the case of not finding their destinations within the search area. This phenomenon indicates that in real systems, perhaps hub nodes are extensively utilized and afford heavy traffic flow.

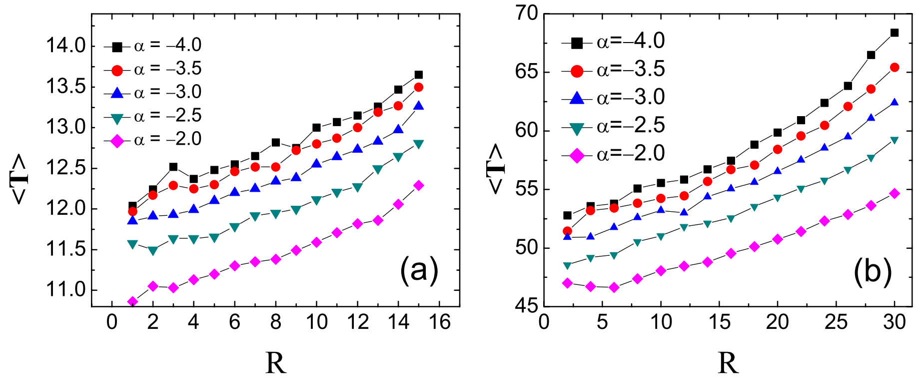

Fig. 3. Average packet travel time $\langle T \rangle$ versus R for (a) $N = 1000$ and (b) $N = 5000$ for different α. The other parameter is $C = 1$.

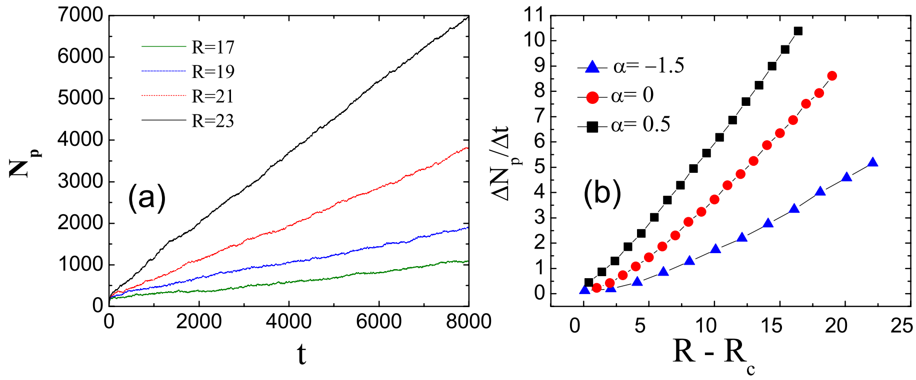

Fig. 4. (a) The evolution of traffic load with time for different R in the congested state. (b) The increasing speed of load $\Delta N_p/\Delta t$ versus $R - R_c$ with different α. N is set to 1000 and $C = 1$.

Decreasing α in a PNNN strategy allows packets to circumvent the heavily loaded nodes. Although the network capacity reaches an upper limit when α is lower than a specific value, considering the contribution of hub (large capacity) nodes to the high transmission speed, further reducing α will indeed affect the average packet travel time $\langle T \rangle$. Thus by studying $\langle T \rangle$ in the case of maximal network capacity, the optimal α can be sought out. Figures 3a and 3b show the $\langle T \rangle$ versus R for different α lower than -2 with $N = 1000$ and $N = 5000$, respectively. It is found that in the whole range of R, the system demonstrates the smallest $\langle T \rangle$ by choosing $\alpha = -2$, and the lower the value of α, the larger the $\langle T \rangle$. Therefore, we can conclude that $\alpha = -2$ is the optimal choice for both maximizing R_c and minimizing $\langle T \rangle$. In addition, compared to Figure 2a, we find that the PIA rule shifts the optimal value of α from 1.5 to 2.0.

Studying traffic properties in the jammed state is also meaningful for alleviating traffic congestion. Figure 4a shows the evolution of traffic load i.e. N_p versus t with different R. In the cases of $R > R_c$, all the curves follow linear functions and the distinction among them is the increasing speed of accumulated packets within the network. In Figure 4b, we report the increasing speed of traffic load with changing R for different α. When R does not approach the phase transition point R_c, three curves display linear functions of $R - R_c$ and the slopes of them are all less than 1 and decrease with reducing α. On one hand, a slope of less than 1 indicates that although the system enters the jammed state, not all the $R - R_c$ packets are piled up per step in the network. On the other hand, small slope corresponding to small α implies that packets are more easily accumulated on the congested nodes in the case of larger α. The phenomenon of slope less than 1 also suggests that when R is not too large in the congested state, the congested nodes in the network only take the minority, while most of the other nodes can still work. Therefore, the congestion of the system can be alleviated just by enhancing the handling and delivering capacity of small quantities of severely congested nodes.

When the generating rate approaches the critical point R_c, the load series exhibits some interesting properties. A big fluctuation is observed near the critical R_c. This phenomenon is called temporary crises in reference [22], and the huge load will dissipate over a relatively long time period. Under the condition of the steady state, the load series shows almost no fluctuations. When R exceeds R_c, the balance of the number of created and removed packets is destroyed and the load increases linearly with time as depicted in Figure 4a. The big fluctuation near the

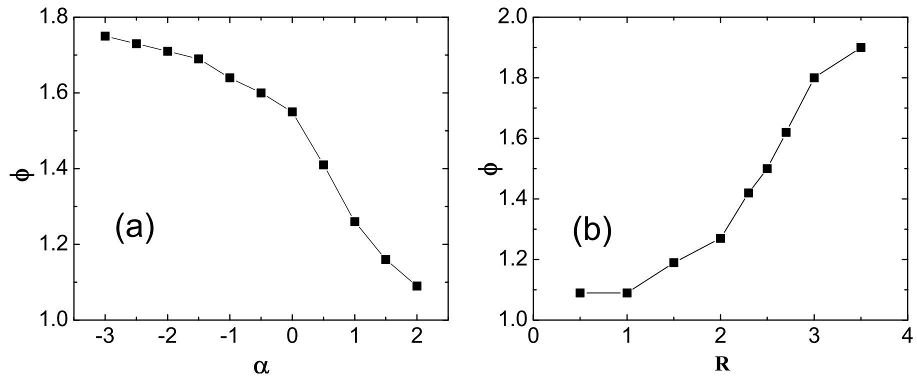

Fig. 5. The slope ϕ of the load time series spectrum as a function of (a) α with fixed $R = 1$ and (b) generating rate R with fixed $\alpha = 2$. Other parameters are $N = 3000$ and $C = 1$.

phase transition point reflects the self-regulating ability of the system. The severe transient local congestion corresponding with the climax of the fluctuation can still be evacuated after a considerable long period of time. Moreover, we observe that the extent of the big fluctuation strongly depends on α. It is found that the biggest fluctuation appears when α is approximately equal to 1.5. Small α according to equation (1) results in the redistribution of heavy load from the large degree nodes to the small ones. However, the big fluctuation near the critical point is mainly due to the contribution of the large degree nodes with severe traffic burdens. Therefore, for small α, the extent of fluctuation is less than that of large α. Otherwise, for very large α, the quite low network capacity which can be seen in Figure 1b induces the small fluctuation values. Due to the competition of these two effects, the largest fluctuation is obtained by choosing $\alpha = 1.5$.

The features of power spectrum of load series are discussed in detail. In real traffic systems such as computer networks and highway traffic, $1/f$-like noise is observed as a common property. The $1/f$ noise denotes that the power spectrum varies as a power-law $S(f) \sim f^{-\phi}$ with the slope $\phi = 1$. The spectrum exponent ϕ characterizes the nature of persistence or the correlation of the load series. $\phi = 2$ indicates zero correlation associated with Brownian motion; $\phi > 2$ indicates positive correlation and persistence i.e., if the process was moving upward (downward) at time t, it will tend to continue to move upward (downward) at future times t'; $\phi < 2$ represents negative correlation and anti-persistence. Therefore, the exhibition of $1/f$ noise of real traffic systems indicates the characters of negative correlation and anti-persistence. The power spectrum $S(f)$ vs. f of load series in our model is investigated and we find the slope ϕ of the spectrum is not only correlated with generating rate R but also with parameter α. The simulations show that for large α, $1/f$-like noise emerges and the system shows the anti-persistence property. With decreasing α, the slope ϕ reduces, ultimately to 2 when α reaches the flat range of Figure 1b. The disappearance of the correlation is mainly caused by the regularity of preferentially forwarding packets to the smaller degree nodes for small α. This routing strategy leads to the averaging of traffic load on all the nodes, and therefore alleviates the traffic burden of large degree nodes and improves the capacity of the network. This rule weakens the effect of hub nodes on which the packet destinations will be found with high probability and more packets can go directly toward their destinations. When the packet density on the large degree nodes is high, the packets will be quickly delivered and removed from the network and contribute to the decrease of the traffic load. However, the hub nodes only take the minority, which leads to the fluctuation of packet density on them, therefore, the property of anti-persistence of load series is attributed to the scale-free structure of BA model and the routing strategy in the condition of large α which increases the packet density on the hub nodes.

The slope ϕ of the spectrum as a function of generating rate R is also investigated. We find that the negative correlation of load series is destroyed by increasing R. For large R, heavy traffic goes on the network and packets are accumulated on some nodes. The queues are permanently formed on the nodes with heavy traffic burden and usually these nodes are large degree nodes with a large search area. The formed queues weaken the randomness of the packets density on those nodes, therefore, as mentioned above, the feature of anti-persistence disappears. Figures 5a and 5b report the behavior of ϕ as functions of α and R, respectively.

The position of phase transition point R_c influenced by the nodes forwarding capacity C, link density of BA network m and network size N is also studied. Figures 6a and 6b show that the network capacity is strongly improved for small α with increasing C and m. As we have mentioned, small α leads to the average of the load among all the nodes, hence, enhancing the individual node capacity intensively increases the entire network capacity and the system shows better performance in the case of small α. High link density can decrease the average packet travel time. Suppose that if the network is fully connected with links existing among any pair of nodes, then each packet can arrive at its destination during one time step. However, increasing m according to the preferential attachment mechanism of the BA model also enhances the connectivity of the maximal degree node who bears the

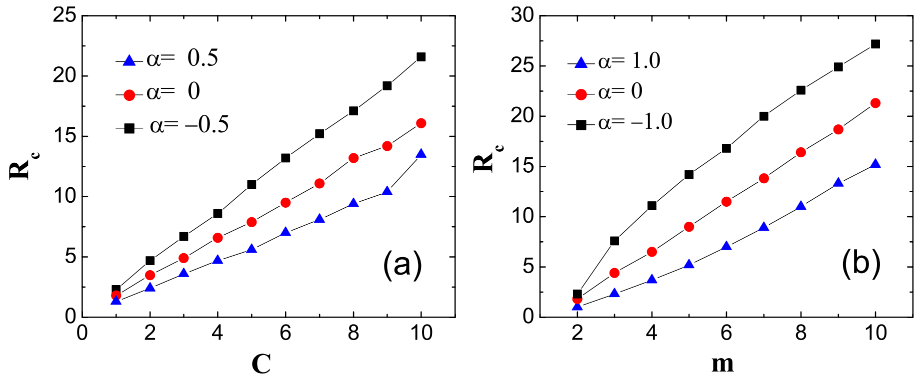

Fig. 6. The critical rate R_c versus (a) individual node capacity C with fixed $m = 2$ and (b) link density parameter m of BA model with $C = 1$ for different α. N is set to 1000.

heaviest traffic burden and determines the network capacity for large α. These two effects can explain the behavior shown in Figure 6b. The plot of R_c versus network size N displays some interesting phenomena as shown in Figure 7. When α is not small, R_c is not a monotonous function of N. This phenomenon may be ascribed to the interplay of three factors. The first one is that each node has its own forwarding capacity C and a larger network size contains larger quantities of nodes which naturally enhance the network capacity. The second factor is the increment of average path length of the network topology with expanding size N, which results in the longer average packet travel time and therefore reduces the network capacity. The third factor is the larger maximal degree corresponding with the larger network size. As discussed above, the larger maximal degree following our PNNN strategy for the same C will decrease the capacity of the system. Perhaps in the medium network size, R_c is mainly affected by the second and third factors, and in the other range, the first factor becomes a dominator. Therefore, the non-monotonous behavior when α is not small is observed. As to the situation of small α, due to the load averaging effect, the third factor is negligible, and compared with the first factor, the second factor is not important because the increment of average path length is very slow with increasing network size. Hence, for small α R_c is an increased function of N.

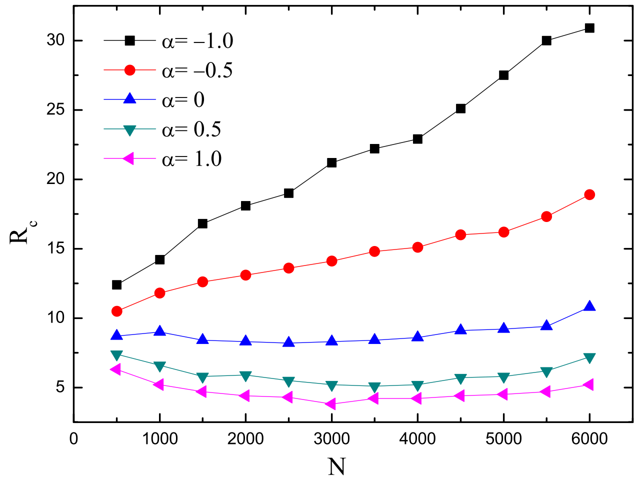

Fig. 7. R_c versus network size N for different α with $C = 1$.

4 Conclusions

We introduce an efficient traffic routing strategy enlightened by the so-called NNN search strategy. The low cost of acquiring the local NNN information makes the strategy possess potential application. By adjusting parameter α, the scale-free network can reach its maximal handling and delivering capacity and the new strategy has proved its efficiency for routing data packets. Traffic properties including packet travel time distribution and evolution of traffic load series are investigated in detail. We find that the extent of the huge fluctuation appearing near the critical point is correlated with α. Moreover, it is found that the anti-persistence property of load series not only depends on the generating rate but also on the parameter α. For large α or small R, traffic load displays the $1/f$-like noise behavior which is confirmed with empirical observations of computer networks. The condition under which the $1/f$-like noise is observed also indicates that the hub nodes bear a severe traffic burden and the volume of traffic flow is far from the maximal capacity of the networks in real traffic systems. Moreover, we find that increasing individual node capacity and the link density of the underlying structure can considerably improve the capacity of scale-free networks, especially in the case of small α. The network capacity influenced by the network size N is also discussed and some interesting behavior is found. Our strategy may be useful for designing communication protocols for large scale-free networks due to the low cost of local information requirement and the strongly improved network capacity.

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