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Agreement dynamics of finite-memory language games on networks

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Abstract. We propose a Finite-Memory Naming Game (FMNG) model with respect to the bounded rationality of agents or finite resources for information storage in communication systems. We study its dynamics on several kinds of complex networks, including random networks, small-world networks and scale-free networks. We focus on the dynamics of the FMNG affected by the memory restriction as well as the topological properties of the networks. Interestingly, we found that the most important quantity, the convergence time of reaching the consensus, shows some non-monotonic behaviors by varying the average degrees of the networks with the existence of the fastest convergence at some specific average degrees. We also investigate other main quantities, such as the success rate in negotiation, the total number of words in the system and the correlations between agents of full memory and the total number of words, which clearly explain the nontrivial behaviors of the convergence. We provide some analytical results which help better understand the dynamics of the FMNG. We finally report a robust scaling property of the convergence time, which is regardless of the network structure and the memory restriction.

PACS. 89.75.-k Complex systems – 05.65.+b Self-organized systems – 89.65.Ef Social organizations; anthropology

1 Introduction

In the past few years, statistic physics was deemed important for understanding the collective social behavior in systems consisting of adaptive agents. Typical examples include opinion formation, origin and evolution of languages and the dynamics of evolutionary games [1,2]. By means of simple models, some interesting self-organized behaviors have been observed. For instance, in the language dynamics, a collective agreement on naming objects could emerge among agents via local communications without global coordination [3]. In evolutionary games, high-level cooperation can emerge and persist even though defection action leads to higher payoffs of selfish agents [4]. These interdisciplinary issues have drawn a lot of interest from various scientific communities [5].

Early studies have mainly focused on the cases that agents either are able to interact with all other agents or occupy nodes of regular lattices. These studies, although not always consistent with real situations, can be considered as the groundwork in the process of understanding the dynamics of real systems. Recently, due to the rapid development of complex networks, it has been found that nontrivial "small-world" and "scale-free" topological properties are shared by many real-world networks [6,7]. Hence, it is natural to consider dynamics on networks with these kinds of features. Understanding the influence of network structures on dynamics has been considered an important issue in many interdisciplinary fields [8].

In this paper, we study the evolutionary dynamics of language based on the Naming Game (NG) model, which is inspired by the field of semiotic dynamics, a new area focusing on the development of shared communication systems composing of multiple agents [9]. A typical example of such systems is the so-called Talking Heads experiment [10], in which a robot assigns names to objects observed through cameras and negotiates with other robots about these names. Recently, models of semiotic dynamics have exhibited practical implication in a new type of web tools, such as del.icio.us and www.flickr.com, through which web users share information by tagging items like pictures and web-sites [11]. The NG, as a model of communicating agents reaching the global consensus through local interactions, can well characterize the origin, spreading and convergence of words in a population. However, because the NG can achieve the ultimate global consensus from a multi-opinion state, which is apparently different from other opinion models [12], it's better to regard the NG as an independent modeling approach for opinion formation in a self-organized system.

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Recently, a minimal version of the NG was proposed by Baronchelli et al. [13]. This model simplifies the original NG model but can as well reproduce the same experimental phenomena. Further work generalized the minimal NG to lower-dimensional lattices [14] and other complex networks [15–18]. Based on the minimal NG, we present a modified NG with respect to the finite memories (inventories) of agents, which takes the bounded rationality of agents into account. We consider several kinds of networks representing the relationships among agents as well, including random networks, small-world networks and scale-free networks. We focus on the time duration that the system is needed to reach the global consensus. The fast convergence in naming objects is of great practical importance in communication systems, not only for the information sharing among agents, but also for saving resources in information storage (such as web servers of the new type of web tools). Interestingly, we found that by tuning the connectivity density, there exists shortest convergence time for all the three types of networks in the Finite-Memory Naming Game (FMNG). Our findings differ from previously reported results as there is no restriction of agents' memories in which fully connected networks result in the fastest convergence [16]. We also did theoretical predictions for better understanding the dynamical properties of the FMNG, like the success rate in negotiation, maximal memory length of agents, and the correlation between the maximal memory length and the number of agents of full memories. Our analytical results are in good accordance with simulations. We finally studied the scaling properties of the convergence time and the maximum number of total words as the network size increases. Simulation results show that the convergence time shows a nonlinear scaling property, regardless of the network topology and the memory restriction.

We organize the paper as follows. In Section 2, we describe the rules of the FMNG and models for generating the required complex networks. In Section 3, we extensively study the dynamics of the Naming Game for both infinite and finite memory cases, which help explain the non-monotonic behavior of convergence in the latter case. In Section 4, we study the scaling properties of the convergence time and the maximum total number of words. In Section 5, we conclude the present work and discuss the relations between the NG and other models.

2 The model

We first build some network structures for our study. We adopt the Erdös-Rényi (ER) model [19] to generate random networks. In the ER model, there is a parameter P, governing the probability of the connection between any pair of vertices. The dependence of the average degree $\langle k \rangle$ on P is $\langle k \rangle = NP$, where N is the number of vertices. We generate small-world networks by adopting the Newman-Watts (NW) model [20], which is a modified version of the original Watts-Strogatz model [21]. In the NW model, a parameter P_{NW} determines the fraction of edges being randomly added to a regular ring graph. Here, the coordination number of the ring graph is 2, so that the NW network possesses fewer triangular structures. Besides, we generate scale-free networks by adopting the Barabási-Albert (BA) model [22]. At each time step, a new vertex is added with m edges being preferentially attached to the existing network. The average degree of the BA network is $\langle k \rangle = 2m$.

In the following, we describe the evolutionary rules of the FMNG model. Each site of a network is occupied by an agent, thus the number of agents is equal to the network size. N identical agents observe single object and try to communicate its name with the others. Each agent is endowed with an internal memory (inventory) to store a number of words. The memory length L is a tunable parameter. When L tends to be unlimited, our model reduces to the original minimal NG model. In FMNG, initially, each agent has an empty memory and the system evolves as follows:

(i) At each time step, a speaker i is chosen at random and then i randomly chooses one of its neighbors as the hearer. This is referred to be the directed NG [16,18].

(ii) If the speaker i's memory is empty, it invents a new name for the object and records it; otherwise, if i already knows one or more names, i randomly chooses one name from its memory for the object. After that, the invented or selected name is transmitted to the hearer j.

(iii) If the hearer j already has this name in its memory, the negotiation is successful, and both agents preserve this name and cancel all other names in their memories; otherwise, the negotiation fails. In the latter case, if the memory of the hearer is not full, the new name will be included in the hearer's memory without canceling any existing names; else if the hearer's memory is full of names, with probability 0.5, the new name transmitted from the speaker will randomly replace one existing name in the hearer's memory; with probability 0.5, nothing happens to the hearer.

It shall be noted that the NG models can be considered as belonging to the class of opinion formation models, but they considerably differ from each other in the number of selectable options of agents. For the Voter model [23–25], each agent has only two options, while for the NG, before reaching the final consensus, an agent can remember a large number of different names for the same object. It is also worth noting that our FMNG model considers only one single object, while in reality, agents can observe a set of different objects. This is due to the assumption that the semantic correlation among objects is neglectable, so different objects can be deemed independent of each other in assigning names to simplify the modeling significantly.

3 Simulation and analytical results of collective properties

We first study the most important quantity, the convergence time T_c defined as the time for reaching the finial consensus, in the FMNG over several kinds of networks.

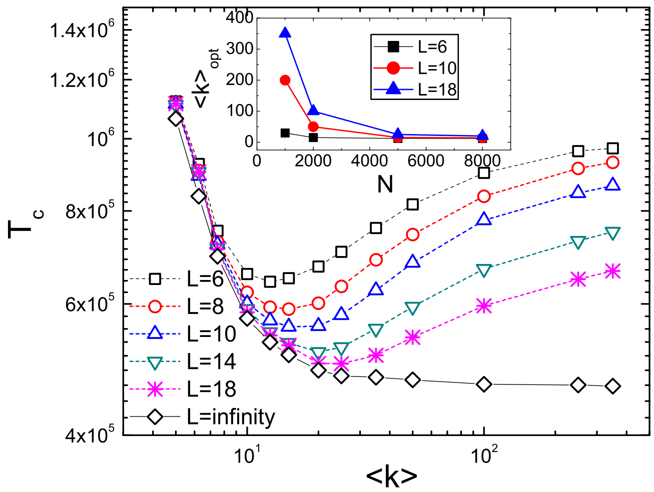

Fig. 1. (Color online). Convergence time T_c as a function of average degree $\langle k \rangle$ in ER random networks for different memory lengths L. The $\langle k \rangle$ can be tuned by network parameter P. The inset is the optimal average degree $\langle k \rangle_{opt}$ as a function of network size N for different memory length L. The network size N is 5000. Each data point is obtained by averaging over 1000 different simulations for each of the 10 network realizations.

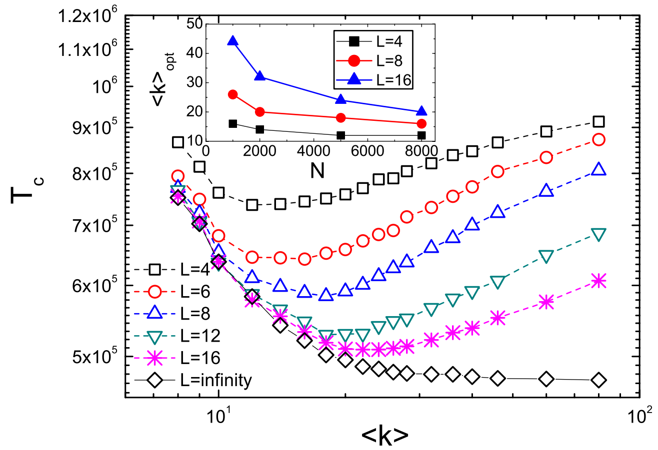

Fig. 2. (Color online) Convergence time T_c as a function of average degree $\langle k \rangle$ in NW small-world networks for different memory lengths L. The $\langle k \rangle$ can be tuned by adding edges to a regular ring graph. The inset is the optimal average degree $\langle k \rangle_{opt}$ as a function of network size N for different memory length L. The network size N is 5000. Each data point is obtained by averaging over 1000 different simulations for each of the 10 network realizations.

The network size N is 5000 for all simulations in this section. As shown in Figure 1, the convergence time T_c is not a monotonic function of average degree $\langle k \rangle$, with a shortest T_c in the middle range of the memory length L. Hence, there exists an optimal $\langle k \rangle$ corresponding to the fastest convergence. Similar phenomena can be observed in NW and BA networks, as displayed in Figures 2 and 3, respectively. Here, for the NW network, $\langle k \rangle$ can be tuned by adding different numbers of edges [20]. For the BA network, $\langle k \rangle$ is controlled by the parameter m while preserving the exponent of the power-law degree distribution [22]. This interesting non-monotonic behavior is contrary to previously reported results in the minimal NG [14–16]. As shown in Figures 1–3 for the minimal NG without memory restriction (L = infinity), T_c is a monotonically decreasing function of $\langle k \rangle$, that is, the mean-field type (fully connected) networks result in the fastest convergence. In contrast, in our FMNG model, the finite memory effect induces the existence of an optimal connectivity density among agents for all the three types of networks. Another phenomenon in Figures 1–3 that should be noticed is that the value of the optimal $\langle k \rangle_{opt}$ has dependence on the memory length L, i.e., the higher values of the L, the larger of the optimal $\langle k \rangle$. In the large limit of L, the dependence of T_c on $\langle k \rangle$ will approach a monotonic behavior with no optimal $\langle k \rangle$, which is consistent with the case of no memory restriction studied previously. The dependence of the optimal average degree $\langle k \rangle_{opt}$ on network size N is shown in the insets of Figures 1–3. As N increases for all three types of networks, $\langle k \rangle_{opt}$ decreases and the decrement speed becomes more and more slower.

In order to explain the finite-memory effect on the consensus achievement, we need to study evolutionary properties of some basic quantities, that is, the total number of names in the system $N_w(t)$ and the average rate of success $S(t)$ in negotiation. Before studying the finite memory case, we start with the simpler case of agents with infinite-memory lengths. Recent works have extensively studied the dynamics of the minimal NG, not only in fully-connected networks [14] but also in homogenous and heterogenous networks [15,16,18]. A very useful analytical result is given for the evolution of $S(t)$ at the early stage, by assuming the success rate between two connected agents to be proportional to the probability of choosing the edge between them [16]. Since in the initial stage, most memories are empty, the repetition of interactions contributes to the success rate $S(t)$. We deem that this viewpoint can be generalized to the stable stages after the transient stage. Simulation results on $S(t)$ for ER, NW and BA networks are shown in Figures 4a$_1$–4c$_1$, respectively. One can find that after a short period, $S(t)$ reaches a plateau, which is almost independent of time. Whereafter, $S(t)$ quickly increases to 1 and all agents reach the finial agreement in naming the object. We argue that in the stable range, the success of an interaction is still determined by the repetition of interactions. If we assume that each agent i has interacted typically with only one neighbor j, the probability for the repetition of such an interaction is

$$\frac{1}{N}\left(\frac{1}{k_i}+\frac{1}{k_j}\right). \tag{1}$$

Neglecting the degree correlation between vertices i and j, and summing over all nodes i and j in the network, the success rate in the stable range can be obtained as

$$S_f=\sum_{i,j=1}^{N}\frac{1}{N}\left(\frac{1}{K_i}+\frac{1}{k_j}\right)=\left\langle\frac{2}{k}\right\rangle. \tag{2}$$

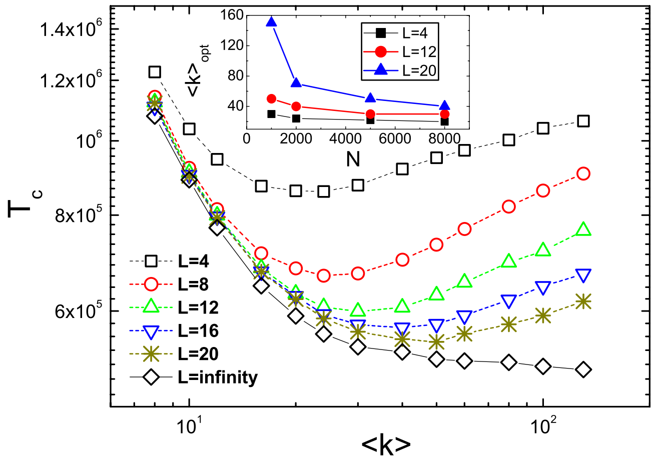

Fig. 3. (Color online). Convergence time T_c as a function of average degree $\langle k \rangle$ in BA scale-free networks for different memory lengths L. The $\langle k \rangle$ is controlled by the number of edges new vertices attached to the existent network. The inset is the optimal average degree $\langle k \rangle_{opt}$ as a function of network size N for different memory length L. The network size N is 5000. Each data point is obtained by averaging over 1000 different simulations for each of the 10 network realizations.

For ER and NW networks, due to the homogenous degree distributions, we have

$$S_f \simeq \frac{2}{\langle k \rangle}. \tag{3}$$

Simulation results on ER and NW networks are in good accordance with predictions depicted in Figure 4a$_2$ and 4b$_2$, respectively, which validates the analytical results based on some assumptions.

In the case of BA networks, due to the heterogenous structural property, $\langle \frac{1}{k} \rangle \neq \frac{1}{\langle k \rangle}$. Hence, we should consider the degree distribution to calculate the dependence of S_f on $\langle k \rangle$. Note that

$$S_f = \left\langle \frac{2}{k} \right\rangle = 2 \int_{k_{min}}^{k_{max}} \frac{1}{k} P(k) dk, \tag{4}$$

where $P(k) = \sigma k^{-3}$ for BA networks [22], with $\sigma = k_{min} \times \langle k \rangle$, which can be easily obtained from

$$\int_{k_{min}}^{k_{max}} \sigma k^{-3} k dk = \langle k \rangle. \tag{5}$$

Substituting σ into equation (4) yields

$$S_f \simeq \frac{2}{3k_{min}^2} \langle k \rangle = \frac{8}{3\langle k \rangle}, \tag{6}$$

where $k_{min} = \langle k \rangle / 2$. In Figure 4c$_1$, $S(t)$ for different $\langle k \rangle$ shows similar evolutionary behavior in BA networks as that in ER and NW networks, i.e., there is a flat behavior in the middle range of t. The values of $S(t)$ in this range are well predicted by equation (6), as shown in Figure 4c$_2$.

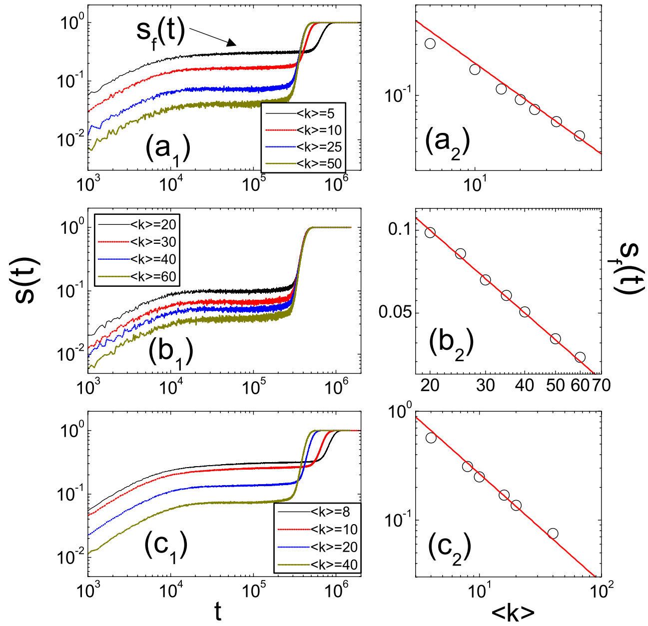

Fig. 4. (Color online). The evolution of success rate $S(t)$ in (a_1) ER random networks, (b_1) NW networks and (c_1) BA networks in the infinite memory case. (a_2), (b_2) and (c_2) are the comparison of simulations results and theoretical predictions (the lines). The network size N is 5000. Data points are obtained by averaging over several thousand runs. $S_f(t)$ results from the average over the data points in the flat range of $S(t)$.

Next, we turn to the evolutionary behavior of the total number of names in the system $N_w(t)$. Simulation results of $N_w(t)$ on ER, NW and BA networks are shown in Figure 5a$_1$–5c$_1$, respectively. One can find that in these figures, there exist maximum values of the total memory N_w^{max} for different $\langle k \rangle$, and the larger $\langle k \rangle$, the higher N_w^{max}. Below, we focus on the correlation between N_w^{max} and $\langle k \rangle$ and try to provide some theoretical results. In the evolutionary process, there are two factors contributing to the change of $N_w(t)$: one is the success of a negotiation in an interaction between two agents, which can result in the deletion of names in both agents; the other is the failure, which can result in one name included into the hearer's memory. We adopt the mean-field approximation, i.e., assuming the memory of each agent is approximately N_w/N, the evolution of N_w is expressed as

$$\frac{dN_w(t)}{dt} = -S(t) \left(\frac{2N_w(t)}{N} - 2 \right) + (1 - S(t)), \tag{7}$$

where the first term on the right is the contribution of the success, while the second one is that of the failure. To acquire a solution of equation (7), the expression of $S(t)$ is required. Due to the complex behavior of $S(t)$, which possesses three types of features, it is not easy to predict such evolution. Fortunately, by combing Figures 4 and 5, we found that N_w^{max} emerges when $S(t)$ stays in the stable range with slight changes. The values of $S(t)$ in this range for different $\langle k \rangle$ have been analytically calculated, so that by substituting these obtained results into equation (7), the dependence of N_w^{max} on $\langle k \rangle$ for distinctive networks

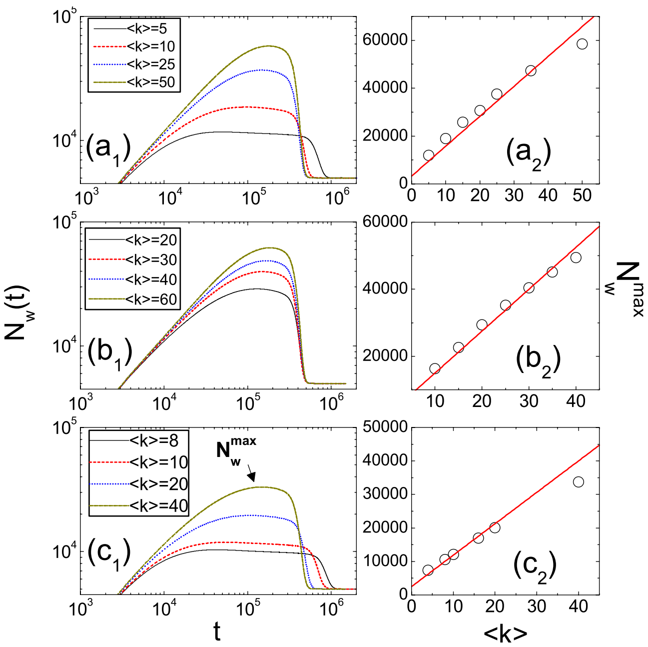

Fig. 5. (Color online). Total number of names $N_w(t)$ in (a_1) ER random networks, (b_1) NW networks and (c_1) BA networks in the infinite memory case. (a_2), (b_2) and (c_2) are the comparison of simulations results and theoretical predictions (the lines). The network size N is 5000. Data points are obtained by averaging over several thousand runs.

can be achieved. For ER and NW networks, considering the extremum condition for $N_w(t)$, i.e.,

$$\left.\frac{dN_w(t)}{dt}\right|_{N_w(t)=N_w^{max}} = 0, \tag{8}$$

and by combing equation (3), we obtain

$$N_w^{max} = \frac{N}{2}\left(1+\frac{\langle k\rangle}{2}\right). \tag{9}$$

In Figures 5a$_2$ and 5b$_2$, we give a comparison between the above prediction and simulation results. They match well for not-so-large $\langle k\rangle$. Similarly, for the BA network,

$$N_w^{max} = \frac{N}{2}\left(1+\frac{3\langle k\rangle}{8}\right). \tag{10}$$

As shown in Figure 5c$_2$, numerical simulations well confirm the prediction for not-so-large $\langle k\rangle$ in BA networks. In the case of very large $\langle k\rangle$, many triangular structures emerge, which enhance the success rate in negotiations. When three agents form a triangle, if a name is transmitted from agent i through its neighbor j to its neighbor's neighbor l, then the interaction between i and l will more easily succeed. Hence, the presence of clustering structures leads to some differences between our theoretical results and simulation results. A detailed discussion about the effects of clustering structures on the dynamics of the NG was given in reference [26]. Moreover, one can observe that for BA networks, the differences between predictions and simulations are more apparent than that of ER and NW networks. This is due to the fact that memories of agents are not identical in heterogenous networks; instead, the memory has some positive correlation with agent's degree, as reported in reference [16]. Thus, assuming that all agents have about the same number of names in their memories by our mean-field approximation leads to larger differences.

After we have identified a clear relationship between dynamical properties of the main quantities and the average degree in the infinite memory case, we now study the dynamical behavior of the FMNG. Compared to the NG, the evolution of the FMNG is more complicated. A stable value S_f not only is a function of $\langle k\rangle$, but also depends on the memory length L. We first perform simulations of S_f depending on L for different $\langle k\rangle$ in ER, NW and BA networks, with results shown in Figures 6a$_1$–6c$_1$, respectively. There is one common feature in these figures, that is, the restriction of memory length reduces the values of S_f, compared to the infinite memory case. When S_f does not change for large L (S_f reaches a platform), it indicates that the dynamics are not affected by the memory restriction and the FMNG reduces to the minimal NG. The dash lines are the theoretical estimations for the infinite memory case. Since there are slight differences between simulation results and theoretical estimations for the infinite memory case (as shown in Fig. 4), the simulation results of S_f with no memory restriction (at the platforms) may be slightly higher or lower than the estimations, as shown in Figure 6. The phenomenon that the memory restriction reduces S_f can be easily explained, since the decrease of the number of names in each agent will naturally decrease the number of shared names in each pair of connected agents, leading to the decrease of the success rate in negotiations. With the increase of L, the influence of memory restriction becomes weaker and at last S_f approaches the predicted value in the infinite memory case.

Similar to the analysis of the total number of names $N_w(t)$ for the infinite-memory NG, for the FMNG we can write the following evolution equation:

$$\frac{dN_w(t)}{dt} = -S(t)\left(\frac{2N_w(t)}{N}-2\right)+(1-S(t))\frac{N-N_L}{N}, \tag{11}$$

where N_L is the number of agents with full memory, i.e., the memory length is L. As one can see, the above equation differs from equation (7) only in the last term on the right-hand side. In the FMNG, when a negotiation fails, a new name will be included into a hearer's memory if the hearer's memory is not full; while if a hearer's memory is full of names, a new name will randomly replace an old one, or be dropped, which doesn't contribute to the change of $N_w(t)$. The last term of equation (11) represents the contribution of those agents without full memories to the evolution of $N_w(t)$. Using the extremum condition (8), we obtain

$$N_L = N - \frac{2S_f(N_w^{max}-N)}{1-S_f}. \tag{12}$$

Here, we cannot get any theoretical result about S_f due to its complexity. By substituting simulation results into the

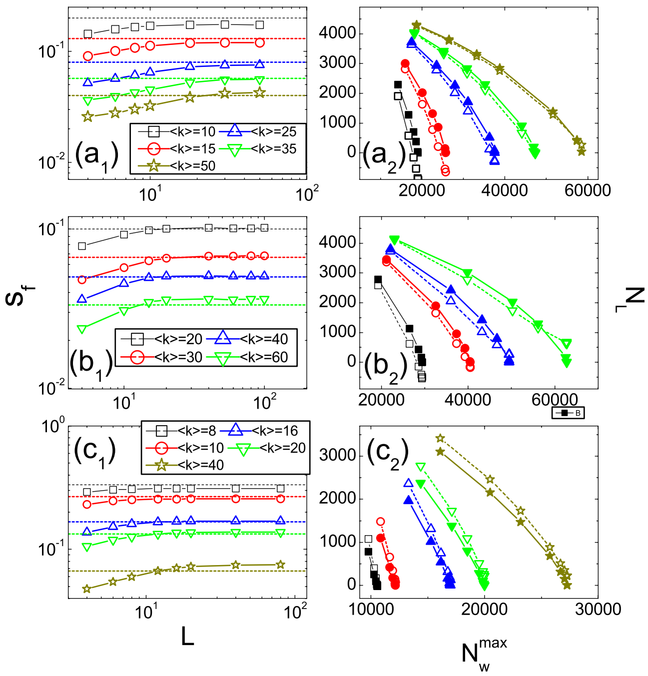

Fig. 6. (Color online). Left panels: S_f as a function of memory length L in (a_1) ER random networks, (b_1) NW networks and (c_1) BA networks. The dash lines are predictions for the infinite memory case. Right panels: the number N_L of agents with full memory as a function of the maximum total names N_w^{max} in (a_2) ER, (b_2) NW and (c_2) BA networks. The hollow symbols are rough estimations for comparison. Data points are averaged over several thousand runs with $N = 5000$.

above equation, the correlation between N_L and N_w can be acquired. A comparison of simulation results and analytical results is shown in Figures 6a$_2$–6c$_2$, respectively. Each curve is for a given $\langle k\rangle$. The L of data points in each curve from the top to the bottom corresponds to that of data points from the left to the right in the right panels. These results indicate that N_L has a negative correlation with N_w^{max} for the same $\langle k\rangle$, but it is a nonlinear correlation. On the other hand, for the same L, larger $\langle k\rangle$ means more agents of full memories. These phenomena can be easily understood by noticing that in the original minimal NG, larger $\langle k\rangle$ induces larger N_w^{max}. Thus, for the same degree of memory restriction L, more agents of full memories emerge for larger $\langle k\rangle$. Moreover, for identical $\langle k\rangle$, because N_w^{max} is the same in the infinite memory case, higher degree of memory restriction leads to more full-memory agents, hence more reductions in N_w^{max}.

All the above results help explain the non-monotonic behavior of the convergence time T_c versus $\langle k\rangle$ in the FMNG. We know that in the infinite memory case, N_w^{max} is proportional to $\langle k\rangle$. Hence, for small $\langle k\rangle$, the memory restriction almost has no influence on the FMNG and the behavior of T_c in the FMNG is similar to that of the original minimal NG, i.e., with the increase of $\langle k\rangle$, T_c decreases correspondingly. On the other hand, for large $\langle k\rangle$, the system demands a large total memory of agents to quickly reach the consensus. In this case, the finite L strongly affects the dynamical behavior of the FMNG. An agent with degree k receives different names from all its neighbors. Averagely the number of shared names between an agent and one of its neighbors can be roughly estimated by $N_w(k)/k$, where $N_w(k)$ denotes the number of names recorded by an agent of degree k without memory restriction. For a finite L, in particular when $L < N_w(k)$ (the agent's memory is full), the number of shared names becomes L/k. Thus, the success rate in the interaction of the agent with its neighbors decreases since the success rate is proportional to the number of shared names. Moreover, for the same L, a larger $\langle k\rangle$ leads to a lower success rate. The reduction of the success rate also depends on the number of full-memory agents: the more the full-memory agents, the lower the success rate. In Figure 6, on the right panels, we have observed that for the same L, the number of full-memory agents for larger $\langle k\rangle$ is much more than that for smaller $\langle k\rangle$, so the success rate is reduced by the increment of $\langle k\rangle$, inducing longer T_c. From the above analyses, we can conclude that in the large $\langle k\rangle$ range, T_c is positively correlated with $\langle k\rangle$. Combining the phenomena for both small and large limits of $\langle k\rangle$, there should exist an optimal value of $\langle k\rangle$ in its middle range, resulting in the fastest convergence.

Another observed phenomenon is that as L increases, the optimal $\langle k\rangle$ moves in the abscissa toward larger values. This can also be explained by noticing that weakening the memory restriction enlarges the range of $\langle k\rangle$, in which memory effects have no influences on T_c. Hence, the decreasing area of T_c becomes broader and the optimal $\langle k\rangle$ values increase.

4 Scaling properties

Previously reported results in reference [16] have demonstrated that the convergence time and the maximum memory in the original NG scales with the size of networks. Interestingly, these scaling properties are not affected by the topological properties, such as the average degree, the clustering and the particular degree distribution. In this section, we focus on how the memory restriction influences the scaling law. Is it independent of the memory length or does the scaling behavior disappear?

We study the scaling property of our FMNG on ER, NW and BA networks, respectively. In these networks, the average degree is tunable. Hence, we explore the combing effects of both the average degree $\langle k\rangle$ of each network and the finite memory length L on the scaling properties of convergence time T_c and the maximum total number of words N_w^{max}. We first fix $\langle k\rangle$ for each network to obtain the dependence of T_c and N_w^{max} on the network size N. Figure 7a–7c show T_c as a function of N for different L on ER, NW and BA networks, respectively. One can find that the scaling property is preserved in the FMNG. We have checked that the value of the scaling exponent is 1.4 ± 0.2, which is regardless of the memory length L for all considered networks. The obtained exponent is also consistent with that in the original NG presented in reference [16], which indicates that the scaling law of the convergence

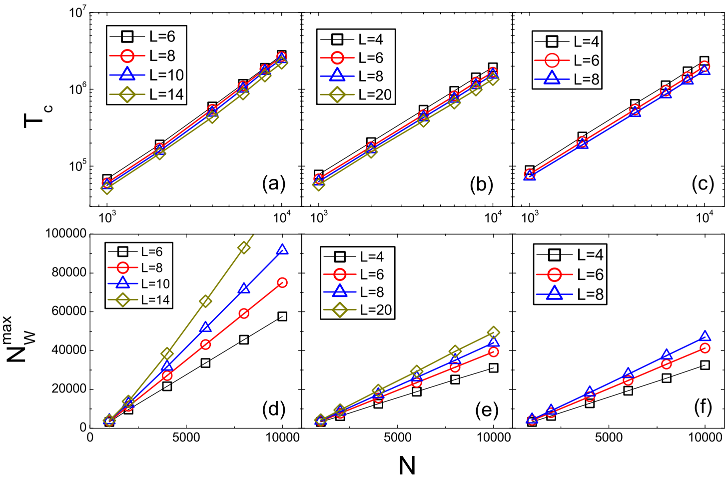

Fig. 7. (Color online.) The convergence time T_c as a function of the network size N for different memory length L on (a) ER, (b) NW and (c) BA networks. The maximum total number of words N_w^{max} as a function of N for different L on ER, NW and BA networks are shown in (d), (e) and (f), respectively. The parameter P in ER networks is fixed to 0.015, so the average degree $\langle k \rangle = 0.015N$. The average degree of NW and BA networks are fixed to $\langle k \rangle = 16$ and $\langle k \rangle = 30$, respectively. Each data point is obtained by averaging over 1000 different simulations for each of the 10 network realizations.

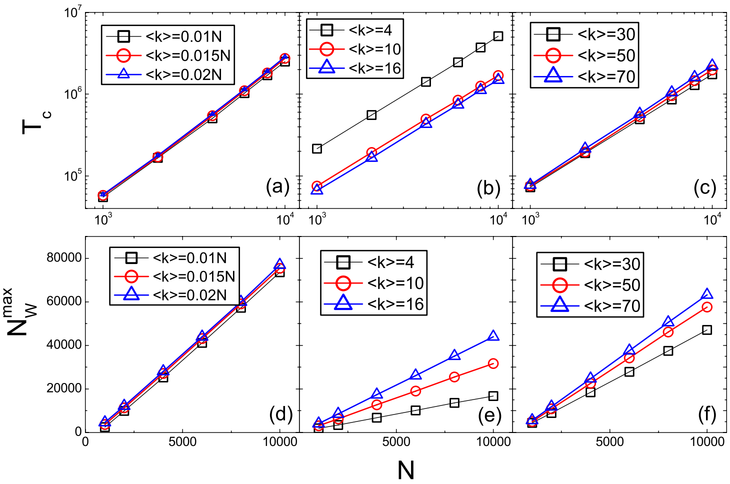

Fig. 8. (Color online.) The convergence time T_c as a function of the network size N for different average degrees on (a) ER, (b) NW and (c) BA networks. The maximum total number of words N_w^{max} as a function of N for different average degrees on ER, NW and BA networks are shown in (d), (e) and (f), respectively. The memory length L is fixed to 8 for all networks. Each data point is obtained by averaging over 1000 different simulations for each of the 10 network realizations.

time is a general robust feature regardless of both the topological details and the memory length. Although the scaling exponent is a general feature for different conditions, the detailed convergence time differs as L varies. It shows that longer memory length correspond to shorter convergence time. Figures 7d–7f report the maximum total number of words N_w^{max} as a function of N for ER, NW and BA networks, respectively. Similar to the original NG, N_w^{max} scales linearly with the size of the network, but the increase speed is positively correlated with L.

Next, we fix the memory length and vary the average degree $\langle k \rangle$ of considered networks to see whether the robust scaling property is affected by $\langle k \rangle$ in the FMNG. As shown in Figures 8a–8c, the convergence time T_c still scales as $N^{1.4}$ for different $\langle k \rangle$ on ER, NW and BA networks. These results indeed demonstrate that the convergence time in the FMNG still follows a universal scaling property, independent of the topological features of networks and the memory length of agents. The maximum total number of words N_w^{max} shows analogous linear scaling (Figs. 8d–8f) with the network size, and the larger of $\langle k \rangle$, the higher values of N_w^{max}.

5 Conclusion

In this paper, we have studied the dynamics of a new model of the Finite-Memory Naming Game over three types of representative complex networks. The finite memory takes into account the fact of bounded rationality of agents or the finite resources for storing information in communication systems. We have found an interesting phenomenon, that is, by tuning the average degree of the network, there exists an optimal average degree leading to the fastest convergence, which is contrary to the previously reported results in the original infinite-memory minimal Naming Game model, where the mean-field type networks result in the fastest consensus convergence. In order to explain such non-monotonic behavior in the Finite-Memory Naming Game, we first considered the infinite memory case by means of simulating the evolutionary behavior of some main quantities, including the success rate in negotiation and the total number of names in the network. We then carried out corresponding analysis, which are in good accordance with simulations. Whereafter, based on the obtained results, we studied the effects of memory length on the success rate, as well as the relation between the maximum total number of names and the number of agents with full memories. We gave a rough estimation for this relation. With all obtained results, we finally explained the emergence of the optimal value of the average degree corresponding to the shortest convergence time. We further investigate the convergence time and the maximum total number of words depending on the network size. We found a robust scaling property of the convergence time, which is not affected by the network topology and the memory restriction of agents. Our work reveals that the finite-memory effect plays a significant role in modeling the dynamics of games such as language evolution in communication systems, with foreseeable practical importance.

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