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Spatial Games Based on Pursuing the Highest Average Payoff *

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We propose a strategy updating mechanism based on pursuing the highest average payoff to investigate the prisoner's dilemma game and the snowdrift game. We apply the new rule to investigate cooperative behaviours on regular, small-world, scale-free networks, and find spatial structure can maintain cooperation for the prisoner's dilemma game. In the snowdrift game, spatial structure can inhibit or promote cooperative behaviour which depends on payoff parameter. We further study cooperative behaviour on scale-free network in detail. Interestingly, non-monotonous behaviours observed on scale-free network with middle-degree individuals have the lowest cooperation level. We also find that large-degree individuals change their strategies more frequently for both games.

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Cooperative behaviour is ubiquitous in many biological, social and economic systems.[1] Yet, understanding the emergence and persistence of cooperation remains a challenge to many natural and social scientists.[2] So far, evolutionary game theory has provided a common mathematical framework to characterize and investigate the evolution of cooperation.[3−5] The prisoner's dilemma game (PDG) and the snowdrift game (SG), as general models, are often used in this field. In the original games of PDG and SG, two players simultaneously decide whether to cooperate or defect. They both receive R upon mutual cooperation and P upon mutual defection. If one chooses defection and the other chooses cooperation, the defector will get T and cooperator receives S. In the PDG, the rank of the four payoff values is $T > R > P > S$, so defection is the best strategy regardless of the opponent decision. While in the SG, the order of P and S is exchanged, such that $T > R > S > P$. Thus, in the SG, the best action now depends on the opponent: to defect if the other cooperates, but to cooperate if the other defects.

Since the pioneering work of Nowak and May,[6] many interests have been given to the effect of spatial structures, such as regular graphs[7−11] and complex networks[12−32] on cooperative behaviour. Many interesting phenomena have been observed in structured games. A surprising finding is that cooperation is often inhibited by the spatial structure in the SG,[8] which is in sharp contrast to one's intuition, since the SG favours cooperation compared to the PDG. Another important finding is that scale-free networks provide a unifying framework for the emergence of cooperation.[16] Very recently, Wang *et al.*[32] find that there exist a discontinuous phase transition and hysteresis loops in structured games.

In the evolutionary game, players update their strategies according to certain rules. 'Tit-for-tat',[33] 'Win–Stay, Lose–Shift',[12] and stochastic evolutionary rule proposed by Szabó *et al.*[12] are commonly used rules. Apart from these, players can adopt death–birth mechanism,[28−30] self-questioning mechanism,[18,27] global payoff-based mechanism,[32] and other mechanisms to update strategies. It is well accepted that the updating rule plays an important role in the evolution of cooperation. In this Letter, we propose a structured game model based on pursuing the highest average payoff: a player will switch his strategy with some probability if his average payoff is not the highest among his neighbours and himself.

Consider that N players are placed on the nodes of a certain network. In every round, all pairs of connected players play the game simultaneously. In each time step, each player gets his total payoff by playing the game with all his immediate neighbours. As used in Refs. [17,19,26], we evaluate the success (or fitness) of the players by their average payoffs: total payoff divided by their connectivity degree. In the next time step, each player changes his current strategy to his opposite strategy with probability

$$W_i = (\max(i) - P_i)/D, \qquad (1)$$

where P_i is the average payoff of player i, $\max(i)$ is the highest average payoff among all player i's neighbours and player i himself, $D = T - S$ for the PDG and D

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$= T - P$ for the SG. Following previous studies, we set $T = b$, $R = 1$, $S = 0$, $P = 0$, where $1 < b < 2$ for PDG; $T = 1 + r$, $R = 1$, $S = 1 - r$, $P = 0$, where $0 < r < 1$ for SG. Hence each game is controlled by a single payoff parameter, b for PDG and r for SG.

We aim to explore the cooperative behaviour influenced by this new updating mechanism, which has not been considered so far. Different from the stochastic evolutionary rule introduced by Szabó *et al.*,[12] players do not adopt neighbours' strategies, instead they take self-questioning mechanism and only use payoff information coming from themselves and their neighbours.

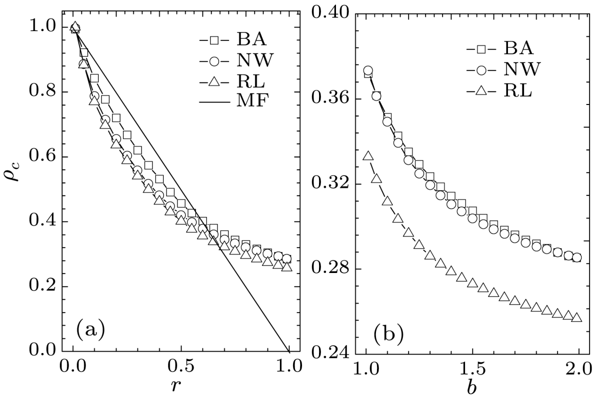

Fig. 1. The density of cooperators ρ_c, on MF, RL, NW, BA networks. For RL, NW, BA networks, average connectivity $\langle k \rangle = 8$. Each data point is obtained by averaging over 20 individual realizations, network size $N = 10000$. Left panel: ρ_c as a function of r for SG. Right panel: ρ_c as a function of b for PDG.

The key quantity for characterizing the cooperative behaviour is the density of cooperators ρ_c, which is defined as the fraction of cooperators in the whole population. We study ρ_c on four typical networks: the fully connected network (mean-field case, MF), regular lattices (RL) with periodic boundary conditions, Newman–Watts small world network (NW)[35] and scale-free Barabási–Albert network (BA).[36] In our simulations, ρ_c is obtained by averaging from last 5000 Monte Carlo (MC) time steps of total 10000 MC time steps, where the system has reached a steady state. In the initial states, cooperators and defectors are uniformly distributed among all the players. For MF, we can provide analytical results for ρ_c. When the system is stable, namely, no individuals change their strategies anymore, the payoff of any cooperator should be equal to that of defector. Hence for SG, we have

$$\rho_c + (1 - \rho_c)(1 - r) = \rho_c(1 + r), \quad (2)$$

where the left side is the average payoff of cooperator, and the right side is the average payoff of defector. From Eq. (2), $\rho_c = 1 - r$. For the PDG, one can write a similar equation:

$$\rho_c = \rho_c b, \quad (3)$$

which gives $\rho_c = 0$.

From Fig. 1, one can observe that for both games, BA network can best promote cooperation, while RL network has the lowest cooperation level. The difference is caused by the heterogeneity of network. As is well known, RL network is an extremely homogeneous network while BA network is heterogeneous. For SG, ρ_c on three spatial networks (BA, NW, RL networks) are lower than $1 - r$ expected in well mixed populations when r is small ($r < 0.6$). By contrast, spatial structure can enhance cooperation for large r ($r > 0.6$). This phenomenon is opposite to the previous results,[10] in which authors found that spatial structure favours cooperation when r is small while inhibits cooperation when r is large. Among many previous studies on structured PDG, cooperators often die out when b is large, while cooperation can be persistent over the entire range of b on all three spatial networks using the new updating rule.

In the following, we focus on cooperative behaviour on BA network. It is found that connecting degree plays an important role in structured games with the new updating rule. As shown in Fig. 2, for SG, large-degree individuals have lower ρ_c than that of small-degree individuals when r is small ($r = 0.1$). Interestingly, when r becomes large ($r = 0.9$), ρ_c is non-monotonous function of connecting degree k with the middle-degree individuals having the lowest cooperation level. Similar nontrivial phenomenons are observed in the PDG over the entire range of b ($b = 1.1$, $b = 1.5$, $b = 1.9$ respectively).

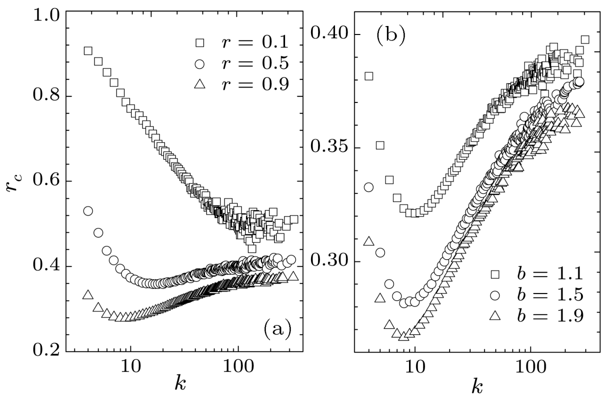

Fig. 2. The density of cooperators ρ_c, as a function of connecting degree k on BA network. Network size $N = 10000$, average degree $\langle k \rangle = 8$. Each data point is obtained by averaging over 20 individual realizations. Left panel: $r = 0.1$, 0.5, 0.9 for SG. Right panel: $b = 1.1$, 1.5, 1.9 for PDG.

In order to understand non-monotonous behaviour induced by connecting degree, it is necessary and interesting to investigate the frequency of varying strategy f_v, which is defined as the proportion of varying strategy times in total time steps. Higher value of f_v corresponds to change strategy more frequently, and vice versa. If players keep their strategies unchanged through total time steps, then $f_v = 0$. By contrast, if players change strategies at each time steps, then $f_v = 1$. From Fig. 3, one can observe that f_v increases with r increases, but almost is independent of b. In addition, larger-degree individuals are more frequent to change strategies for both games. For larger-degree individuals, although they earn more from more neighbours compared to lower-degree individuals, the average payoffs obtained from each neighbour in general will not be the highest, and may even much less than the highest payoff. In contrast, if a small-degree individual defects and its neighbours at a certain time step are just all cooperators (the probability of this occurrence will be higher than that of higher-degree individuals), then the small-degree defector will have the highest average payoff. From this perspective, smaller-degree individuals have higher probability to gain the highest payoffs, so that tend to be inactive in strategy updating; while higher-degree individuals change strategies more frequently. It is interesting to note that high value of f_v can inhibit or enhance cooperation for large-degree individuals in SG. When r is small, changing strategies frequently means large-degree individuals can not keep cooperation for enough time, so they can not have high cooperation level. By contrast, when r is large, high value of f_v makes large-degree individuals have approximately equal probability to take defection or cooperation, which prevents cooperation level from falling to very low value. Individuals prefer to take defection in PDG because defection usually bring higher payoff. Compared with small-degree individuals, middle-degree individuals can keep defection more easily since they have relatively steady number of cooperative neighbours. On the other hand, middle-degree individuals change strategies less frequently than large-degree individuals. As a result, middle-degree individuals have the lowest cooperation level. Similar case occurs in SG when r is large.

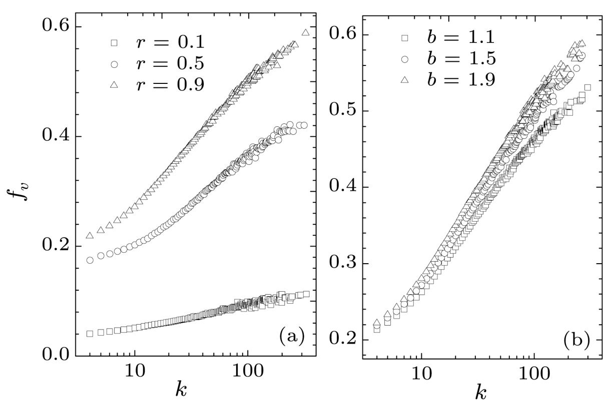

Fig. 3. Frequency of varying strategy f_v, as a function of connecting degree k on BA network. Here f_v is obtained from the last 5000 MC time steps of total 10000 MC time steps of evolution. Left panel: $r = 0.1$, 0.5, 0.9 for SG. Right panel: $b = 1.1$, 1.5, 1.9 for PDG. Other parameters are the same as those in Fig. 2.

In conclusion, we have studied the prisoner's dilemma game and the snowdrift game based on a new updating rule, in which a player will change his strategy to opposite strategy with some probability if his average payoff is not the highest among his neighbours and himself. In the snowdrift game, we find that spatial structure inhibits the cooperative behaviour for small payoff parameter r, contrarily promotes the cooperation when r is large. For the prisoner's dilemma game, cooperation can be persistent over entire range of payoff parameter b on spatial networks, such as regular, small-world, and scale-free networks. In particular, non-monotonous behaviours are observed on scale-free network with middle-degree individuals have the lowest cooperation level for large r and entire range of b. In addition, we find that large-degree individuals are more frequent to change their strategies for both games. Our work may be helpful in understanding the role of strategy updating mechanism in the evolutionary games.

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