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Buying on Margin and Short Selling in an Artificial Double Auction Market

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Abstract Leverage trading, which consists of short selling and buying on margin, has been introduced into stock markets in many countries, including China. Ever since, there have been heated debates on how leverage trading influences financial markets. In this paper, an agent-based artificial market model is developed to simulate market behaviors and to analyze the influence of the leverage ratio on liquidity, volatility and price-discovery efficiency. In our artificial market, heterogeneous agents submit limit orders based on the fundamentalist or chartist strategy, and their effective supplies and demands can be increased by short selling or margin trading. Numerical analyses are performed in both one-sided and two-sided markets. We find that in one-sided markets, leverage trading can increase market liquidity and volatility, and decrease price-discovery efficiency. However, in the two-sided market, the increase of liquidity is much smaller, the volatility is decreased, and the price-discovery efficiency is improved. Generally, this model provides some meaningful results, which are supported by many other studies, and these findings underscore the necessity of building up a two-sided market when introducing leverage trading into stock markets.

Keywords Agent-based model · Continuous double auction · Short selling · Margin trading

1 Introduction

Leverage trading meets the desires of investors to gain greater profits. It consists of short selling, namely that investors can use their cash and shares as collateral and

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borrow stocks from securities companies to sell, and buying on margin, namely that investors can borrow money to buy more stocks. Leverage trading enlarges supply and demand, which can increase the liquidity of the financial market and potentially further affect market prices and volatility. Leverage trading has been controversial throughout the academic circles and the practical realm. Supporters believe that the introduction of leverage trading helps improve liquidity and pricing efficiency and promotes the formation of a risk-sharing mechanism in financial systems. However, critics argue that leverage trading can amplify risks, increase market volatility and even lead to market crash. In 1934, in order to control market volatility and to protect traders from excessive speculation, the Securities and Exchange Act imposed the initial and maintenance margin requirements. In addition, over the next 40 years, the initial margin requirements were changed 22 times. Under the influence of the US subprime mortgage crisis from 2007 to 2009, many countries, including the US, the UK, France and Australia, took measures to enhance the regulations of leverage trading. Since then, there have been heated debates about whether the margin requirement is an effective method to control risks and maintain market stability. Many efforts have been made to figure out how leverage trading and margin requirements influence the financial market, especially market volatility, pricing efficiency and liquidity.

This paper is organized as follows. Section 2 is literature review and Sect. 3 introduces our agent-based market model with leverage trading. In Sect. 4, we provide an analysis of the simulation results of the model. Finally, in Sect. 5, we present our conclusion.

2 Literature Review

Many scientific studies focus on the impact of margin requirements on price volatility. A series of studies by Hardouvelis are typical examples . Hardouvelis (1988, 1990) and Hardouvelis and Peristiani (1989, 1992) test the relationship between margin requirements and volatility by regression analyses in the US stock market and Japanese stock market; these studies reach the same conclusion: there is a negative correlation between margin requirements and volatility, namely, increasing margin requirements helps reduce volatility. However, Hardouvelis and Theodossiou (2002) then studies the issues in bull and bear markets and finds that the relationships are different in different markets. In a bull market, the negative correlation is obvious, but this correlation does not hold in a bear market. In contrast with these perspectives above, Moore (1966) and Officer (1973) claim that margin requirements do not influence volatility. This issue is still ardently discussed. Recently, Kim and Jung (2013) studies the data from Japanese stock market by using the Component GARCH model and finds that increasing margin requirements can reduce long-run volatility but does not affect short-run volatility. In addition, Chou et al. (2015) shows that when margin requirements are increased, institutional traders leave the stock market more easily than individuals, which will increase market volatility.

Many studies discuss the relationship between leverage trading and price discovery. Greenwald and Stein (1988) states that margin transactions increase liquidity and help push overvalued (undervalued) market prices back to the fair values. Moreover, by analyzing cross-sectional data on market prices, Bris et al. (2007) concludes that relaxing short-sale restrictions increases price-discovery efficiency. On the contrary, (Grossman and Miller 1988; Hsieh and Miller 1990) claim that short selling and margin trading amplify investment risks, exacerbate market volatility, and make market prices dramatically deviate from their fundamental values. Some studies, such as those of Miller (1977) and Duffie et al. (2002), find that forbidding short selling is detrimental to the reflection of negative information, which leads to the overvaluation of securities. Furthermore, Boehme et al. (2006) states that, in Miller's model, overvaluation occurs under two necessary conditions: short-sale constraints and disagreement about the fair value. If the traders do not have different opinions about market information, the market prices will not be overvalued.

As for liquidity, most research results are consistent and suggest that leverage trading can increase the trading volume and increase liquidity. By analyzing the data from 111 stock markets all over the world, Charoenrook and Daouk (2005) takes the turnover rate as a measure of liquidity and concludes that leverage trading can improve market liquidity. Chou et al. (2015) measures liquidity by the bid-ask spread and finds out that increasing margin requirements will reduce trading activity and decrease liquidity. Moreover, numerous empirical studies, including those of Hardouvelis (1990) and Hardouvelis and Peristiani (1992), have produced similar results: high initial margin requirements will reduce the trading volume, especially the number of speculators' transactions.

To sum up, most of those extant studies are empirical research. Due to the use of different data and different analyze methods, the conclusions are not consistent. The debate is still open, and no widely accepted theory exists. An alternative approach is agent-based modeling, which is a research approach that can commendably simulate the real financial market and help us reconstruct and explain the emergent financial phenomenon from the bottom up. In agent-based models, traders can be heterogeneous and have bounded rationality, which is much more consistent with the real financial market. Compared to the theoretically oriented models, much more stylized facts, including volatility clustering and fat-tailed distribution of returns, can emerge through these computationally oriented agent-based models. These agent-based models can be classified into three types according to the methods of describing agents individual behavior patterns. LeBaron et al. (1997, 1999) use genetic algorithm to describe the learning behavior of investors, based on the Santa Fe artificial stock market. The simulation results show complex patterns of actual financial market. Then LeBaron (2001b) uses neural networks to represent the agent learning process and LeBaron (2001a, 2002) perform some extensive calibration exercises based on the model presented in LeBaron (2001b). Besides, there are also some researches focus on the market microstructure instead of the agents' bounded rationality and learning behaviors, using zero intelligent (ZI) agent models. Smith et al. (2003), Farmer et al. (2005) and Mike and Farmer (2008) test the prediction power of their ZI models and the simulations perform well in predictions. Preis et al. (2007) uses ZI artificial stock market and the simulation results can reproduce some stylized facts, such as fat-tailed distribution of the returns and over-diffusive Hurst exponent for medium time-scales. Moreover, there are also some studies use typical strategies models. In these models, agents adopt some typical strategies, such as chartist strategy, fundamentalist strategy and so on, see Chiarella et al. (2006, 2012). All researches mentioned above are models without interactive mechanism. In these models, agents only get information from markets, and their behaviors will not be influence by other agents directly. However, there are also some interactive multiple models (IMM), in which agents can be affected by other agents. For instance, Cont and Bouchaud (2000) uses a percolation model (CB model) to analyze the relationship between the heavy tails distribution of returns and the herding behavior of investors. Hazy and Tivnan (2004) focuses on the organization systems and the agents in their model are connected through networks.

Recently, there are some studies which introduce short-selling regulations to the agent-based model markets to observe the short-selling issues. Mizuta et al. (2015) finds that price limits, blanket short selling regulations, and uptick rules can prevent overshoot and make the market efficiency during a bubble collapse, but the last two will make the market overpriced under the normal situation. Yagi et al. (2010) conducts simulations both in regulated and unregulated artificial markets based on multi-agent and states that the former is more stable while bubbles form in the latter. Similarly, Witte and Kah (2010) states that short selling constraints, such as tick rules, can make the market stable, but they can also cause the market overpriced. Anufriev and Tuinstra (2013) introduces trading cost to the artificial market and finds out that the short selling constraints will not change the local stability, but can increase market volatility when the price is not stable.

In this paper, we explore these issues by building an artificial market model with heterogeneous agents, inspired by the research of Chiarella et al. (2012) (the CHP model) that gives a dynamic analysis of the microstructure of moving average rules in double auction market. In CHP model, agents are learning, and they have bounded rationality and heterogeneous beliefs. They submit orders according to their individual strategies, and the market price is endogenous and formed by the transaction of these orders. The CHP model takes double auction mechanism, which is widely used in real stock market, and performs well in characterize financial stylized facts, such as volatility clustering, insignificant autocorrelations of raw returns, and significant slowly decaying autocorrelations of the absolute returns. Our research focuses on how leverage ratio affects the investment behaviors of heterogeneous investors and further affects the stock market. So we adopt the typical strategies model, which ignores the interactive effects between agents and is easy to be used to analyze the investment behavior. We extend the CHP model by, for instance, allowing traders to submit limit orders that are valid for multiple units of the stock several times a day, and introducing leverage trading.

We simulate trader behaviors and analyze the influence of the leverage trading on liquidity, volatility and price-discovery efficiency. Difference from the most studies, we analyze the issue by changing the leverage ratio, instead of introducing some short selling regulations, such as price limits, blanket short selling regulations, and uptick rules. Besides, we perform numerical analyses in both one-sided markets (namely, only buying on margin or short selling is allowed) and a two-sided market (namely, both buying on margin and short selling are allowed). From the simulation results, we find that when increasing the leverage ratio, the two-sided market is more stable than the one-sided markets. In addition, leverage trading is good for price discovery in the two-sided market, while it makes the market price deviate from the fundamental value in one-sided markets. These results are similar with those of Zhang and Li (2013), except the volatility part. Zhang and Li (2013) adopts market maker mechanism, and use the standard deviation of market prices as the metric of volatility, whereas we use the double auction mechanism, which is widely used in real stock market, and the realized volatility measurement, which can measure the market volatility better.

3 The Model

There are N traders in our artificial market, who adopt the fundamentalist strategy or chartist strategy. They have one chance to enter the market in a random order during each trading period t, and decide to submit limit orders based on their strategies. $p_{t\tau}$ is the stock's price at time $\tau(t < \tau < t + 1)$ during time period t, which will change as soon as transaction occurs. At the end of each trading period, agents cancel their unfinished orders with a probability of φ_c. We consider the time period t as 5 min, which means that the 48 time periods' simulation represents one trading day (4 h). At the end of each trading day, the order book will be cleared.

In our model, buying on margin and short selling are allowed without interest payments, but the trader will forced to close his positions if the maintain requirement is not met. It is important to note that before the debt is paid off, the trader who has already bought on margin can only further buy on margin and cannot sell short. Similarly, the trader who has sold short can only further sell short.

3.1 Trading Strategies

Traders are heterogeneous in our artificial market, and they can trade based on fundamentalist or chartist strategy. In our model, agents submit limit orders in general. However, if a trader has to close his positions, he can only submit a market order (see Sect. 3.2). A limit order is composed of a limit price l and a volume q. If a limit order (l,q) is submitted by agent i, then this trader promises to buy (sell) q units of stock at a specified price l or lower (higher). The heterogeneous agents in our model will decide the order types (buy or sell), the limit prices and the order volumes according to their own strategies.

Fundamentalist Strategy

Fundamentalists believe that stock has fundamental price p_t^*, which is given as follows:

$$p_t^* = p_{t-1}^* \exp(\sigma_f v_t) \tag{1}$$

where $\sigma_f \geq 0$ is a given constant describing the 5 min volatility of the fundamental value, and $v_t \sim N(0, 1)$ subjects to the standard normal distribution. Fundamentalist cannot know the fundamental value p_t^* exactly, so he consider the fundamental price at time τ as

$$p_{t\tau}^* = p_t^*(1 + \Delta_f z_{t\tau}) \tag{2}$$

where $z_{t\tau} \sim N(0, 1)$ is the normal distribution, and $\Delta_f > 0$ is a constant that measures the degree of erroneous estimate. A small Δ_f means that fundamentalists have more information and can estimate the fundamental value accurately.

If the market price exceeds the fundamental price, the stock is considered overestimated; hence, fundamentalists tend to sell stocks, and vice versa. In addition, we assume that the quantity of stocks that fundamentalists want to trade is proportional to the spread between the observed fundamental price $p^*_{t\tau}$ and the current market price $p_{t\tau}$. Above all, the order type $H_{it\tau}$ and the desired order volume $\bar{q}_{it\tau}$ are determined as follows:

$$H_{it\tau} = \mathrm{sgn}(p^*_{t\tau} - p_{t\tau}) \tag{3}$$

$$\bar{q}_{it\tau} = \lfloor \alpha \left| p^*_{t\tau} - p_{t\tau} \right| \rfloor \tag{4}$$

where $H_{it\tau}$ is a sign function which determines the order type ($H_{it\tau} \doteq +1$ means buy, and $H_{it\tau} \doteq -1$ for sell), and $\alpha > 0$ is a constant that measures the fundamentalist's sensitivity to the price spread. Similar to the CHP model, we set the limit price $l_{it\tau}$ to obey a uniform distribution between observed fundamental value and current market price.

$$l_{it\tau} = \begin{cases} U(p_{t\tau}, p^*_{t\tau}) & (H_{it\tau} = +1), \\ U(p^*_{t\tau}, p_{t\tau}) & (H_{it\tau} = -1). \end{cases} \tag{5}$$

Chartist Strategy

Chartists make decisions according to the moving average price, namely, the average of the last D_i steps' prices; thus,

$$m_{it} = \frac{\sum_{j=1}^{D_i} p_{t-j}}{D_i} \tag{6}$$

For each agent, the value of D_i is different, which implies that the length of the time window is individualized. p_t is the price of time step t, and we set it to be the last transection price during the time step t. Chartists believe that if the market price exceeds the moving average price, the market price will continue to rise; hence, they will tend to submit a buy order, and vice versa. Similarly, we assume that the quantity of stocks that a chartist want to trade is proportional to the spread between the moving average price and the current market price. Therefore, the order type $H_{it\tau}$ and the desired order volume $\bar{q}_{it\tau}$ are determined as follows:

$$H_{it\tau} = \mathrm{sgn}(p_{t\tau} - m_{it}) \tag{7}$$

$$\bar{q}_{it\tau} = \lfloor \beta \left| p_{t\tau} - m_{it} \right| \rfloor \tag{8}$$

where $\beta > 0$ is a constant that measures the reaction coefficient for chartists. In addition, similar to the CHP model, the limit price $l_{it\tau}$ is given as follows:

$$l_{it\tau} = \begin{cases} p_{t\tau}(1 + |\Delta_c z_{t\tau}|) & (H_{it\tau} = +1), \\ p_{t\tau}(1 - |\Delta_c z_{t\tau}|) & (H_{it\tau} = -1). \end{cases} \tag{9}$$

where $z_{t\tau} \sim N(0, 1)$ is the normal distribution, and $\Delta_c > 0$ is a constant that measures the aggressiveness of chartists. A larger Δ_c means that chartists are more aggressive and tend to submit buy (sell) orders with the limit prices, which is higher (lower) than the current market prices.

Random Strategy

In addition, we introduce random traders (not belonging to those N traders) into our model. They represent noise traders with zero intelligence or traders who trade for the demand of liquidity. After a fundamentalist or a chartist submits an order, a random trader may enter the market with the probability φ_ε. Random traders submit the buy order or sell order with the same probability. The limit volume q_ε is a random number between three and ten, and the limit price l_ε is close to market price, namely,

$$l_\varepsilon = p_\tau + \sigma_\varepsilon z_{t\tau} \tag{10}$$

where $z_{t\tau} \sim N(0, 1)$ and $\sigma_\varepsilon > 0$ is the volatility of random offset. Importantly, random traders cannot buy on margin or sell short.

3.2 Leverage

The traders determine the limit prices and the amounts of stocks they want to trade according to their own strategies, and if their wealth cannot satisfy their desires, they can buy on margin or sell short.

The assets a_{it} of trader i hold at time period t is defined as $a_{it} = C_{it} + p_t S_{it}$, where C_{it} is the amount of cash; S_{it} is the amount of stock that trader i holds; and p_t is market price of time step t. In addition, the liabilities consist of the cash $\hat{C}_{it}$ and stocks $\hat{S}_{it}$ that he owes. Therefore, the net wealth w_{it} of trader i is $w_{it} = (C_{it} - \hat{C}_{it}) + (S_{it} - \hat{S}_{it})p_t = c_{it} + s_{it} p_t$. The individual leverage ratio of trader i is $L_{it} = a_{it}/w_{it}$.

For the trader without debt, if his wealth cannot satisfy his desires, he can buy on margin or sell short to enlarge his order volume; therefore, the actual order volume $q_{it\tau}$ for a non-debt trader is defined as follows:

If $H_{it\tau} = +1$,

$$q_{it\tau} = \begin{cases} \bar{q}_{it\tau} & (\bar{q}_{it\tau} \leq C_{it}/l_{it\tau}), \\ \min[\bar{q}_{it\tau}, \lfloor C_{it}/l_{it\tau} + (L-1)w_{it}/l_{it\tau} \rfloor] & (\bar{q}_{it\tau} > C_{it}/l_{it\tau}). \end{cases} \tag{11}$$

and if $H_{it\tau} = -1$,

$$q_{it\tau} = \begin{cases} \bar{q}_{it\tau} & (\bar{q}_{it\tau} \leq S_{it}), \\ \min[\bar{q}_{it\tau}, S_{it} + \lfloor (L-1)w_{it}/l_{it\tau} \rfloor] & (\bar{q}_{it\tau} > S_{it}). \end{cases} \tag{12}$$

where L is the initial leverage ratio.

However, as mentioned in the beginning of Sect. 2, before the debt is paid off, the trader who has bought on margin (sold short) cannot sell short (buy on margin); he can only further buy on margin (sell short). Hence, for the trader who has bought on margin already, he cannot increase his sell order volume by short selling. Moreover, before he submits a limit order, the margin requirement must be checked. If his individual leverage ratio L_{it} is larger than the maintain leverage threshold L^* $(0 < L < L^*)$, i.e.,

$$L_{it} = \frac{a_{it}}{w_{it}} > L^* \tag{13}$$

then his position must be liquidated and he can only submit a market order during this trading step. Furthermore, if this trader's net wealth is negative, he will go bankrupt and leave the market.

However, if his individual leverage ratio meets the margin requirements, namely, $L_{it} \leq L^*$, he can submit a limit order, and the actual order quantity is determined as follows:

If $H_{it\tau} = +1$ and the trader's wealth cannot satisfy his desires, namely, $\bar{q}_{it\tau} > C_{it}/l_{it\tau}$, he can buy on margin; thus,

$$q_{it\tau} = \begin{cases} \bar{q}_{it\tau} & (\bar{q}_{it\tau} \leq C_{it}/l_{it\tau}), \\ \min\left[\bar{q}_{it\tau}, \left\lfloor C_{it}/l_{it\tau} + \max\{0, (L-1)w_{it} - \hat{C}_{it}\}/l_{it\tau}\right\rfloor\right] & (\bar{q}_{it\tau} > C_{it}/l_{it\tau}). \end{cases} \tag{14}$$

and if $H_{it\tau} = -1$, the trader cannot sell short, so the actual order volume is

$$q_{it\tau} = \min(\bar{q}_{it\tau}, S_{it}) \tag{15}$$

It is important to note that paying debt $\hat{C}_{it}$ off has precedence when he get cash through selling stocks.

Similarly, for the trader who has sold short already, before he submit a limit order, the margin requirement must be checked. If his individual leverage ratio L_{it} is larger than the maintain leverage threshold L^*, he has to submit a market order and liquidate his position. Moreover, if his net wealth is negative, he will go bankrupt and leave the market. However, if his leverage ratio meets the margin requirements, he can submit a limit order according his strategy.

If $H_{it\tau} = +1$, the trader cannot buy on margin, so the volume that he can actually submit is as follows :

$$q_{it\tau} = \min[\bar{q}_{it\tau}, \lfloor C_{it}/l_{it\tau} \rfloor] \tag{16}$$

and if $H_{it\tau} = -1$, he can enlarge the order volume by short selling, so the volume is given as follows:

$$q_{it\tau} = \begin{cases} \bar{q}_{it\tau} & (\bar{q}_{it\tau} \leq S_{it}), \\ \min\left[\bar{q}_{it\tau}, S_{it} + \left\lfloor \max\{0, (L-1)w_{it}/l_{it} - \hat{S}_{it}\}\right\rfloor\right] & (\bar{q}_{it\tau} > S_{it}). \end{cases} \tag{17}$$

where L is the initial leverage ratio when short selling. Likewise, if the trader get stocks, paying the debt $\hat{S}_{it}$ off will be the priority.

3.3 The Trading Mechanism

The trading mechanism in our artificial market is a continuous double auction (CDA). In the CDA market, an order book exists to record the limit orders that have not been completely traded. The limit orders in the order book are arranged by limit price in descending order. The highest buy order is called the best bid (q_b, l_b), and the lowest sell order is called the best ask (q_a, l_a) because they are most likely to be traded. If a new limit buy order (q', l') is submitted into the market, it will be matched against the best ask, and, if $l' \geq l_a$, the transaction will take place at the price l_a and the trade volume is $\breve{q} = \min(q', q_a)$. In addition, the situation is similar if the new order is a sell order. If the new limit order cannot be completely traded, it will be sorted in the book as well. In our model, the limit order can be repealed with the probability of φ_c at the end of each trading step. However, at the end of each trading day, the book will be cleared. As mentioned in Sect. 3.1, market orders can be submitted in the market. If a new market order is submitted, it will be traded with the best ask or bid until no order exists on the opposite side of the order book or it has been completely traded. If the new market order cannot be traded completely, it will not be sorted and will be cancelled from the market.

3.4 The Switching of Trading Strategies

At the end of each trading period, each trader can switch his strategy according to his realized profit π_{it}:

$$\pi_{it} = (c_{it} - c_{i(t-1)}) + (s_{it} - s_{i(t-1)})p_t \tag{18}$$

Then, trader i will update the profit measure for trading strategies. E^c_{it} is the profit measure of chartist strategy and E^f_{it} is for fundamentalist strategy. Specifically, if a fundamentalist i conducts transactions during day t, he will adjust his profit measure as follows:

$$\begin{cases} E^f_{it} = \eta E^f_{i(t-1)} + (1-\eta)\pi_{it} \\ E^c_{it} = E^c_{i(t-1)} \end{cases} \tag{19}$$

where $\eta (\in [0, 1])$ is a memory parameter. Similarly, a chartist will adjust his profit measure as follows:

$$\begin{cases} E^f_{it} = E^f_{i(t-1)} \\ E^c_{it} = \eta E^c_{i(t-1)} + (1-\eta)\pi_{it} \end{cases} \tag{20}$$

The only thing that needs to be noted here is that, if a trader fails to conduct a transaction during the time step, his profit measure will not be updated. Finally, trader i chooses to be a chartist in the next trading period with the following probability:

$$\rho_{i(t+1)}^{c} = \frac{\exp[\gamma E_{it}^{c}]}{\exp[\gamma E_{it}^{c}] + \exp[\gamma E_{it}^{f}]} \tag{21}$$

where $\gamma > 0$ is a parameter that determines the intensity of switching strategies. Conversely, the probability to be a fundamentalist is $\rho_{i(t+1)}^{f} = 1 - \rho_{i(t+1)}^{c}$.

4 Simulation Results

The following results are an average of 30 simulations with the same leverage threshold value. For each simulation, we run the model with 1200 periods and 1000 agents. The first 96 steps' prices are exogenously given, and they wobble around fundamental prices to initialize the moving averages; therefore, we abandon the next 384 observations to avoid transient effects. The parameters of simulations are listed in Table 1.

We perform the numerical analysis in one-sided markets (namely, only buying on margin or short selling is allowed) and a two-sided market (namely, both buying on margin and short selling are allowed) and increase the initial leverage ratio to observe how the leverage ratio influences the entire stock market.

Table 1 Parameters used in the simulation

Parameter	Value	Description
N	1000	Number of agents
S_{i0}	{1,2,...,9}	Initial stock endowment
C_{i0}	$1000S_{i0}$	Initial cash endowment
p_0	1000	Initial price
p_0^*	990	Initial fundamental value
σ_f	0.001	Volatility of the fundamental value (5 min)
Δ_f	0.0005	Volatility of fundamental price offset
α	0.2	Reaction coefficient for fundamentalists
β	0.12	Reaction coefficient for chartists
D_i	{20,21,...,90}	Length of MA windows
Δ_c	0.0005	Aggressiveness parameter
γ	0.0005	Intensity of switching
η	0.2	Profit-smoothing parameter
φ_c	0.5	Probability of canceling order
φ_ε	0.01	Probability of issuing a random order
q_ε	{3,4,...,10}	Limit volume of a random order
σ_ε	$\sigma_f p_0^*$	Volatility of random offset
L^*	$1.2L$	Maintain leverage threshold

4.1 Stylized Facts

To demonstrate the validity of our model, we record the time series of market prices and returns in one of the simulations with an initial leverage ratio L of one (namely, no short selling or buying on margin), which are shown in Fig. 1a. We verify that the simulation results are consistent with a series of stylized facts, including volatility clustering, fat-tailed distribution, insignificant autocorrelations of returns, and significant slowly decaying autocorrelations of the absolute returns (see Fig. 1).

Besides, we run the simulation 30 times and provide some descriptive statistics of returns. The median and mean value of the maximum and minimum return, volatility, skewness and kurtosis of returns are showed in Table 2. The kurtosis exceeds 3 for half, which confirms again that the returns do not comply with the normal distribution.

4.2 One-Sided Market (Short-Selling Market)

Firstly, we test the model in short-selling markets, where margin trading is forbidden, and increase the initial leverage ratio L from one to two to observe how the liquidity, market volatility and price-discovery efficiency change. Specifically, we use the daily trading volume V, the realized volatility RV and the difference P and absolute

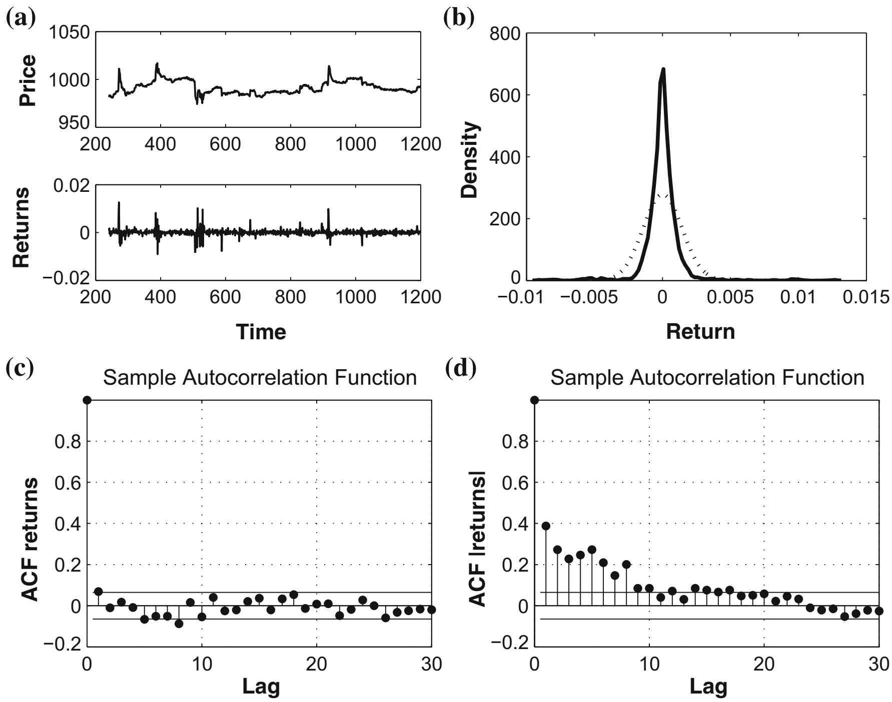

Fig. 1 The time series of market prices and returns (**a**); density of returns (with a normal distribution with same mean and variance) (**b**); autocorrelation of raw (**c**) and absolute (**d**) returns ($L = 1$)

Table 2 Descriptive statistics of the 5 mins' returns (1st panel) and the daily returns (2nd panel) of the simulations of our model ($L = 1$) and the returns of the simulations of the CHP model (3rd panel)

	Max	Min	SD	Skew	Kurt
Median	0.0126	−0.0139	0.0019	−0.1914	20.0904
Mean	0.0592	−0.0157	0.0039	1.2382	56.3941
Median	0.0198	−0.0196	0.0094	−0.0181	3.8371
Mean	0.0416	−0.0414	0.0175	−0.1122	4.2193
Median	0.094	−0.115	0.016	−0.032	9.631
Mean	0.218	−0.247	0.021	−0.537	55.343

difference $|P|$ between the market price and the fundamental value as metrics of liquidity, market volatility and price-discovery efficiency. These indicators are defined as follows:

$$V_d = \sum_{t \in D} V_t \tag{22}$$

$$RV_d = \sum_{t \in D} r_t^2 \tag{23}$$

$$|P|_d = \sum_{t \in D} \left| p_t - p_t^* \right| \tag{24}$$

$$P_d = \sum_{t \in D} (p_t - p_t^*) \tag{25}$$

where D is the set of trading steps in trading day d and $r_t = \ln p_t - \ln p_{t-1}$ is the logarithmic return of each steps (5mins). Hence, the daily trading volume of day d is the sum of trading volume V_t of each trading steps. Besides, we use the 5mins' returns to calculate the realized volatility of day d, considering the fact that realized volatility is a widely used measurement, which is considered to be the unlimited approximation of the integral of instantaneous volatility over the sample interval. As for price-discovery efficiency, we think that if the market prices are close to the fundamental values, the market is efficiency, because fundamental value is considered to be the fair price.

In Fig. 2a, b we can observe that the average trading volume and the realized volatility gradually increase as L increases. This phenomenon implies that the leverage ratio has a positive relationship with market liquidity and volatility, which conforms to the intuitive impression.

Furthermore, Fig. 2c shows that as leverage increases, the average price deviates from the fundamental value, which implies that trading at a high leverage ratio is not conducive to price discovery efficiency. We can also observe that average market price generally drops below the average fundamental value (see Fig. 2d), which is consistent the popular belief that short selling can cause stocks to tumble.

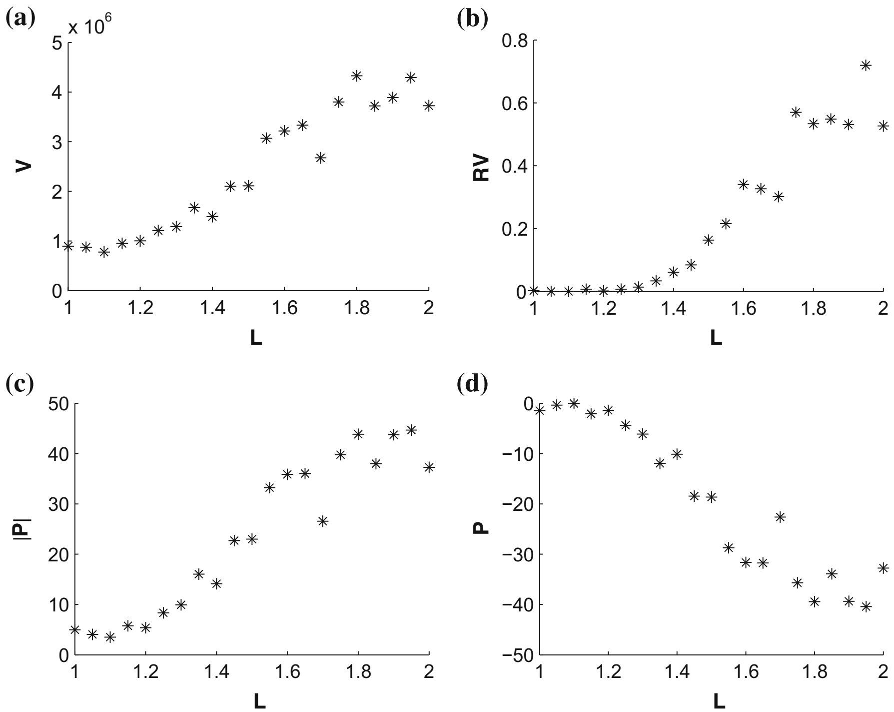

Fig. 2 The results of how (**a**) the trading volume, (**b**) the realized volatility, (**c**) the absolute differences between the average price and the fundamental value, and (**d**) the differences between the average price and the fundamental value change as the leverage ratio L increases in a short-selling market

4.3 One-Sided Market (Margin Trading Market)

We also simulate the model in margin trading market. Contrary to the short-selling market, in the margin trading market, short selling is not allowed. The results are shown in Fig. 3. We only conduct the simulation from $L = 1$ to $L = 1.8$, because in margin trading market, when the leverage ratio beyond 1.8, the market price will become very high and the market will be extremely unstable.

The result about trading volume and realized volatility is similar to short-selling market. As leverage increases, both of them increases (see Fig. 3a, b). But the result about price discovery is a little different from that in short-selling market. Figure 3c shows that as leverage ratio increases, the average price deviates from the average fundamental value, which implies that trading with a high leverage ratio is not conducive to price discovery; however, market prices are generally higher than the fundamental values, which is consistent with the popular belief that margin trading can drive prices up.

4.4 Two-Sided Market

Finally, we perform simulations in the two-sided market, in which both short selling and margin trading are allowed. In two-sided market, the simulation results are quite

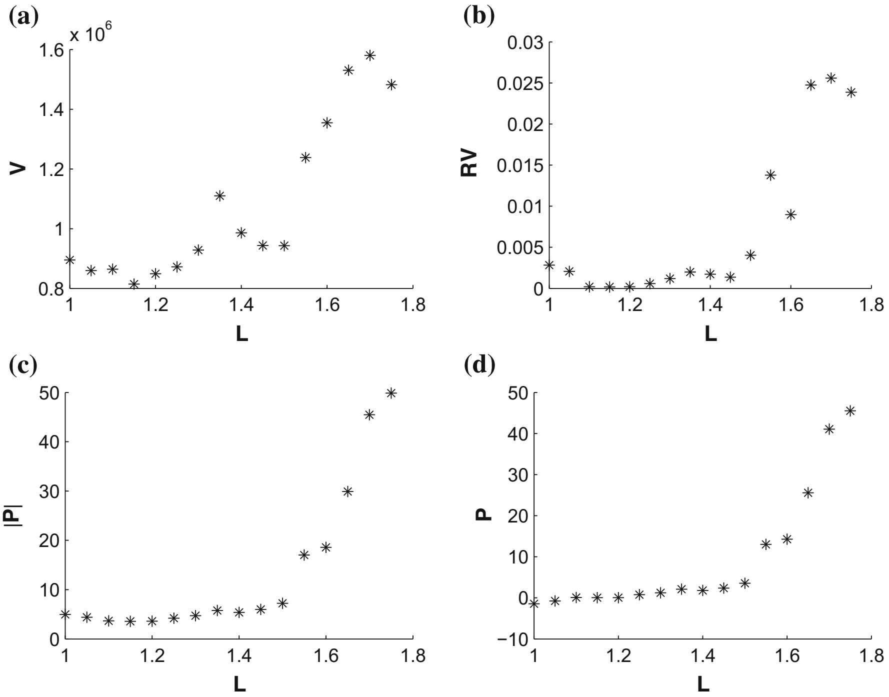

Fig. 3 The results of how (**a**) the trading volume, (**b**) the realized volatility, (**c**) the absolute differences between the average price and the fundamental value, and (**d**) the differences between the average price and the fundamental value change as the leverage ratio L increases in the margin trading market

different. As leverage ratio increases, the average trading volume increases slightly (see Fig. 4a). From the last three subgraph in Fig. 4, we can find out that, once both short selling and margin trading are introduced into the market, the average realize volatility decreases rapidly to a low level and the average price immediately converges on the fundamental value, which implies that leverage trading is conducive to the stability and price discovery in two-sided market. We believe the reason is short selling can causes stocks to tumble and margin trading can drive prices up when they are simultaneously introduced into the market, hence, the market price will be pulled equally in opposite directions and will approach the fundamental price.

4.5 Robust Analysis

In addition, we conduct further simulations in an appropriate parameter space to test the robustness of our model. According to our analysis of the simulation results, we find that α, β and Δ_c are important in our artificial market, which will significantly influence the results. As mentioned in Sect. 3.1, the value of α determines the activity of fundamentalists, who can pull market prices back to the fundamental price and help maintain market stability. Similarly, β determines the activity of chartists, who may push market prices away from the fundamental price and make the market unstable. In

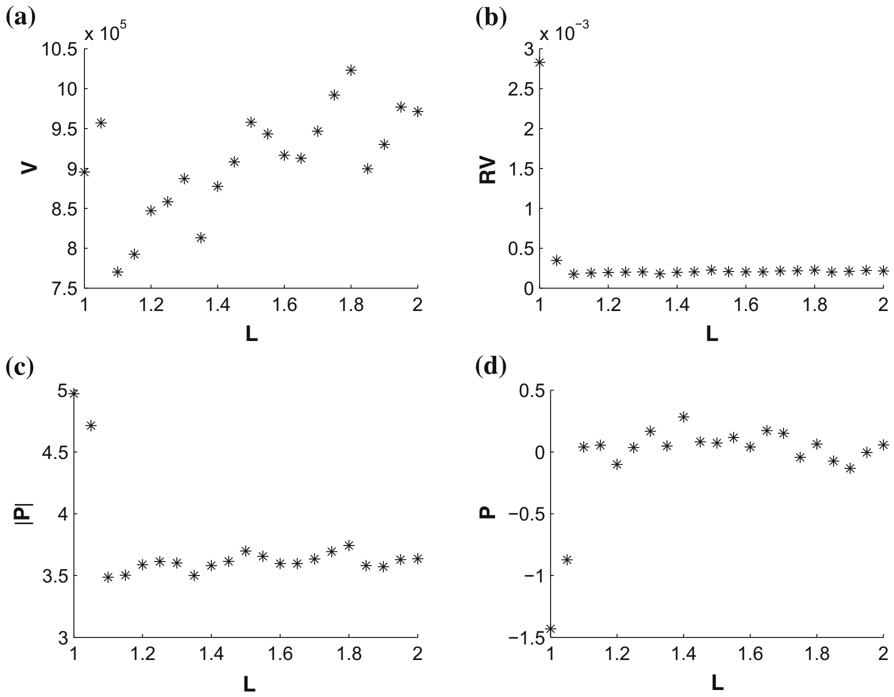

Fig. 4 The results of how (**a**) the trading volume, (**b**) the realized volatility, (**c**) the absolute differences between the average price and the fundamental value, and (**d**) the differences between the average price and the fundamental value change as the leverage ratio L increases in a two-sided market

Table 3 Descriptive statistics of the daily returns of the simulations of our model when change the parameter value

	Max	Min	SD	Skew	Kurt
$\alpha = 0.15$					
Median	0.0244	−0.0365	0.0139	−0.7434	5.5073
Mean	0.0301	−0.1025	0.0285	−1.0407	7.0861
$\beta = 0.1$					
Median	0.0508	−0.0914	0.0371	−0.3916	3.8254
Mean	0.0783	−0.1314	0.0470	−0.4917	5.1687
$\Delta_c = 0.0001$					
Median	0.0207	−0.0222	0.0102	0.1874	3.8075
Mean	0.0453	−0.0444	0.0173	−0.1403	4.6852

addition, Δ_c controls the aggressiveness of chartists, which can reinforce price swings. We offer some basic simulation results under different parameter space in Table 3.

The model presents similar results with $\alpha \in [0.15, 0.30]$, $\beta \in [0.10, 0.20]$ and $\Delta_c \in [0.0001, 0.001]$. If the value of β or Δ_c is below the lower limit, the significant autocorrelations of absolute returns will disappear; if the value exceeds the upper limit, price fluctuations will be violent, and price bubbles or crashes will form. As for α, the opposite results will happen. Moreover, we set the volatility of the fundamental value $\sigma_f \in [0.0004, 0.0007]$ to form the stylized fact of volatility clustering and to avoid the formation of excessive volatility at the same time. γ and η, which are concerned with the switching of trading strategies, can be relaxed to $\gamma \in [0.0001, 0.001]$ and $\eta \in [0.1, 0.9]$. Finally, we increase the number of agents to 5000, and the same qualitative results are found, which proves that the size effect is negligible.

5 Conclusions

In this paper, we build an agent-based artificial market model and focus on the influence of the leverage ratio on trading volume, market volatility and price-discovery efficiency. We perform numerical analyses in one-sided and two-sided markets.

The results show that, as the leverage ratio in one-sided markets increases, the liquidity gradually increases, market volatility increases and the market price dramatically deviates from the fundamental value. Most empirical researches support these results above, which illustrate that, to a certain extent, the separate introduction of margin trading or short selling is not conducive to the market stability and efficiency. However, the results in the two-sided market are quite different from those of the one-sided market. As the leverage ratio increasing, the liquidity increases much more slowly, the market volatility decreases, and the price-discovery efficiency increases, which further supports the conclusion that when leverage trading is introduced, the two-sided market is more stable and efficient than the one-sided market.

Generally, this model provides some meaningful results that are supported by many other studies, and it underscores the necessity of building up a two-sided market when introducing leverage trading into the stock market.

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References

Anufriev, M., & Tuinstra, J. (2013). The impact of short-selling constraints on financial market stability in a heterogeneous agents model. *Journal of Economic Dynamics and Control*, *37*(8), 1523–1543.

Boehme, R. D., Danielsen, B. R., & Sorescu, S. M. (2006). Short-sale constraints, differences of opinion, and overvaluation. *Journal of Financial and Quantitative Analysis*, *41*(02), 455–487.

Bris, A., Goetzmann, W. N., & Zhu, N. (2007). Efficiency and the bear: Short sales and markets around the world. *The Journal of Finance*, *62*(3), 1029–1079.

Charoenrook, A., Daouk, H. (2005). A study of market-wide short-selling restrictions. Available at SSRN 687562.

Chiarella, C., He, X. Z., & Hommes, C. (2006). A dynamic analysis of moving average rules. *Journal of Economic Dynamics and Control*, *30*(9), 1729–1753.

Chiarella, C., He, X. Z., & Pellizzari, P. (2012). A dynamic analysis of the microstructure of moving average rules in a double auction market. *Macroeconomic Dynamics*, *16*(04), 556–575.

Chou, R. K., Wang, G. H., & Wang, Y. Y. (2015). The effects of margin changes on the composition of traders and market liquidity: Evidence from the taiwan futures exchange. *Journal of Futures Markets*, *35*(10), 894–915.

Cont, R., & Bouchaud, J. P. (2000). Herd behavior and aggregate fluctuations in financial markets. *Macroeconomic Dynamics*, *4*(02), 170–196.

Duffie, D., Garleanu, N., & Pedersen, L. H. (2002). Securities lending, shorting, and pricing. *Journal of Financial Economics*, *66*(2), 307–339.

Farmer, J. D., Patelli, P., & Zovko, I. I. (2005). The predictive power of zero intelligence in financial markets. *Proceedings of the National Academy of Sciences of the United States of America*, *102*(6), 2254–2259.

Greenwald, B., & Stein, J. (1988). The task force report: The reasoning behind the recommendations. *The Journal of Economic Perspectives*, *2*(3), 3–23.

Grossman, S. J., & Miller, M. H. (1988). Liquidity and market structure. *The Journal of Finance*, *43*(3), 617–633.

Hardouvelis, G., & Peristiani, S. (1989). Do margin requirements matter? Evidence from us and Japanese stock markets. *Quarterly Review*, *45*, 16–35.

Hardouvelis, G. A. (1988). Margin requirements and stock market volatility. *Quarterly Review*, *45*, 80–89.

Hardouvelis, G. A. (1990). Margin requirements, volatility, and the transitory component of stock prices. *The American Economic Review*, *80*(4), 736–762.

Hardouvelis, G. A., & Peristiani, S. (1992). Margin requirements, speculative trading, and stock price fluctuations: The case of japan. *Quarterly Journal of Economics*, *107*(4), 1333–1370.

Hardouvelis, G. A., & Theodossiou, P. (2002). The asymmetric relation between initial margin requirements and stock market volatility across bull and bear markets. *Review of Financial Studies*, *15*(5), 1525–1559.

Hazy, J. K.,& Tivnan, B. F. (2004). On building an organizationally realistic agent-based model of local interaction and emergent network structure. In *Proceedings of the 36th conference on winter simulation, winter simulation conference* (pp. 1827–1834).

Hsieh, D. A., & Miller, M. H. (1990). Margin regulation and stock market volatility. *The Journal of Finance*, *45*(1), 3–29.

Kim, S., & Jung, T. (2013). The effect of initial margin on long-run and short-run volatilities in Japan. *Journal of East Asian Economic Integration*, *17*(3), 311–332.

LeBaron, B. (2001a). Empirical regularities from interacting long-and short-memory investors in an agent-based stock market. *IEEE Transactions on Evolutionary Computation*, *5*(5), 442–455.

LeBaron, B. (2001b). Evolution and time horizons in an agent-based stock market. *Macroeconomic Dynamics*, *5*(02), 225–254.

LeBaron, B. (2002). *Calibrating an agent-based financial market*. Tech. rep.: Technical report, International Business School, Brandeis University, Waltham, MA.

LeBaron, B., Arthur, W. B., Holland, J. H., Palmer, R., & Tayler, P. (1997). Asset pricing under endogenous expectations in an artificial stock market.

LeBaron, B., Arthur, W. B., & Palmer, R. (1999). Time series properties of an artificial stock market. *Journal of Economic Dynamics and Control*, *23*(9), 1487–1516.

Mike, S., & Farmer, J. D. (2008). An empirical behavioral model of liquidity and volatility. *Journal of Economic Dynamics and Control*, *32*(1), 200–234.

Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. *The Journal of Finance*, *32*(4), 1151–1168.

Mizuta, T., Izumi, K., Yagi, I., & Yoshimura, S. (2015). Investigation of price variation limits, short selling regulation, and uptick rules and their optimal design by artificial market simulations. *Electronics and Communications in Japan*, *98*(7), 13–21.

Moore, T. G. (1966). Stock market margin requirements. *The Journal of Political Economy*, *74*, 158–167.

Officer, R. R. (1973). The variability of the market factor of the new york stock exchange. *The Journal of Business*, *46*(3), 434–453.

Preis, T., Golke, S., Paul, W., & Schneider, J. J. (2007). Statistical analysis of financial returns for a multiagent order book model of asset trading. *Physical Review E*, *76*(1), 801–813.

Smith, E., Farmer, J. D., Ls, G., Krishnamurthy, S., et al. (2003). Statistical theory of the continuous double auction. *Quantitative Finance*, *3*(6), 481–514.

Witte, B., & Kah, C. (2010). Short selling constraints and their effects on market efficiency: Insights from agent-based modeling. In *Eastern economic association annual conference*, Citeseer.

Yagi, I., Mizuta, T., & Izumi, K. (2010). A study on the effectiveness of short-selling regulation using artificial markets. *Evolutionary and Institutional Economics Review*, *7*(1), 113–132.

Zhang, T., & Li, H. (2013). Buying on margin, selling short in an agent-based market model. *Physica A: Statistical Mechanics and its Applications*, *392*(18), 4075–4082.