RBF Network image Representation with Application to CT Image Reconstruction

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Abstract

Radial Basis Function (RBF) neural network can be used as an universal approximator. In this paper, we propose a novel method to apply RBF net to reconstruct 2-dimensional computerized tomography (CT) images from a small amount of projection data. In the method, the cross-sectional image is represented by a RBF network, the unknown cross-sectional image vector is replaced by the function of the network’s weight vector. As proved by us, the line integral of the weight matrix can be calculated providing the projections of the CT image are known. The ART method can be employed to obtain the final reconstructed CT image. Experiments show that the proposed method can obtain the better reconstructed image than the filtered back projection (FBP), and it is also more efficient than ART method alone.

1. Introduction

Image reconstruction techniques of the computerized tomography (CT) are intensively studied for applications to various scientific and engineering problems [1]. Though in some fields the techniques attain a sophisticated level of completion, there are still many issues to be solved or improved. One of them is to develop a method of an image reconstruction from data of a small amount of the projection paths [2][3].

CT image reconstruction algorithm deals with the problem of reconstructing an image from its line integrals. Traditionally, two different kinds of algorithms are employed to reconstruct CT image: one is based on the continuous model and the other is based on the discrete model [1]. The filtered back projection (FBP) is a well-developed method in the first kind[6]. When a sufficient number of high-quality projection data over the whole directions around the object are obtained, the cross-sectional image of the object can be reconstructed rather easily by using FBP. However, in a practical situation encountered in laboratory experiments or field observations, there exist often the problems with insufficient sets of projection data. In these cases, it is very difficult to reconstruct the original image correctly by simply applying the FBP [3]. The Algebraic reconstruction technique (ART) [2][3] which is based on the discrete model can be adopted to solve these problems, but its application is limited by some deficiencies, including a relatively high time complexity and the demand for distribution of the projection paths [6]. So it is worthy for us to study the more effective methods.

In this paper, we propose to adopt radial basis function (RBF) neural network to represent the CT image [9] and to combine ART to reconstruct 2-dimensional CT images from a small amount of projection data.

2. Reconstruction Algorithm

2.1. Reconstruction model

The discrete model of the CT image reconstruction problem can be described as follow: The image to be reconstructed is represented by a square matrix \((X)_{N \times N}\) and the projection is a matrix \((Y)_{L \times M}\). Where \(L\) denotes the number of the given angle and \(M\) denotes the number of the detectors [2]. The projection at a given angle can be calculated as the linear combination of the image pixel at the given direction. Image matrix and the projection matrix can be converted into vector form. Therefore, the problem of reconstruction from projections can then be formulated as the estimation of a column image vector \(X\) that must satisfy the system of approximate equalities.

\[
Y \approx AX .
\]  

\(A\) is a projection process from the \(X\) to \(Y\), now the reconstruction problem is transferred to calculate \(X\). The equation (1) can be written as:

\[
y_j = \sum_{j=1}^{X} a_{i,j} x_j .
\]
2.2 RBF network

RBF neural network (RBFNN) is a most commonly-used feed-forward network [4]. The RBFNN can be expressed as follow:

\[ I(x) = \sum_{i=1}^{N} w_i \phi_i(||x - \mu_i||) \]  

(4)

Where \( x \) denote input vector, \( N \) denote the number of basis functions in RBFNN. \( \mu \) denote the centers of basis function, \( ||\cdot|| \) denote the Euclidean norm. \( w = \{w_i|i = 1, 2, \cdots, N\} \) is the weight vector between the hidden layer and the output layer. \( I(x) = X \) stands for output of the network[10]. Here the Gaussian function \( \phi(x) \) is used as basis function [4][5].

\[ \phi_i(||x - \mu_i||) = \exp \left( -\frac{1}{2\sigma^2} ||x - \mu_i||^2 \right) \]  

(5)

In our work, the cross-sectional image is represented by a RBF network: Coordinates and the cross-section image are the network’s input and the output, respectively. With training data set, equation (4) can be expressed as:

\[ I(x) = \Phi W \]  

(6)

Where \( \Phi \) is the basis function matrix, so the CT image reconstruction problem can be described by \( W \) as:

\[ Y = AX = A\Phi W \]  

(7)

Where \( A \) can be calculated by a line integral of the CT image \( I(x) \), we deduce the equation as follow.

2.3 Computing \( A\Phi \)

In this work, the network plays a role of functional mapping an image coordinate \((x, y)\) to its intensity \(I(x, y)\) as an output. \( x \) and \( I(x) \) are replaced by coordinate \((x, y)\) and its intensity \(I(x, y)\).

\[ I(x, y) = \sum_{i=1}^{N} w_i \phi_i(x, y) = \sum_{i=1}^{N} w_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{||x - \mu_i||^2}{2\sigma_i^2}\right) \]  

\[ = \sum_{i=1}^{N} w_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(x - \mu_i)^2 + (y - \mu_i)^2}{2\sigma_i^2}\right) \]  

(8)

The projection \( p_m \) of the estimated image for the \( m \)th ray can be formulated as equation (9), here, it is assumed that the line equation \( L_m \) of the ray is show in equation (10).

\[ p_m = \frac{1}{L_m} \int_{L_m} I(x, y) d\ell = \sum_{i=1}^{N} w_i H_m^i(x, y) \]

\[ H_m^i(x, y) = \frac{1}{L_m} \exp\left(-\frac{(x - \mu_i)^2 + (y - \mu_i)^2}{2\sigma_i^2}\right) \]  

(9)

Where,

\[ L_m : y = k_m x + b_m \]  

(10)

We assume that the object is located only within a reconstruction area, and thus density value outside the area is totally zero, the integral of infinite range should be almost the same as that of reconstruction area. Thus, changing the integral range as infinite does not make any loss of generalization. Finally, the matrix \( H \) can be obtained from Equation (11) [3].

\[ H_m^i(x, y) = \sqrt{1 + k_m^2} \int_{-\infty}^{\infty} \exp\left[-\frac{(x - c_i)^2}{2\sigma_i^2}\right] + (k_m x + b_m - c_i)^2 / 2\sigma_i^2 dx = \exp\left[-(k_m c_i + b_m - c_i)^2 / 2\sigma_i^2 / (1 + k_m^2)\right] \]  

\[ \times \int_{-\infty}^{\infty} \exp\left[-x'^2 / 2\sigma_i^2\right] dx' = \exp\left[-(k_m c_i + b_m - c_i)^2 / 2\sigma_i^2 / (1 + k_m^2)\right] \]  

(11)

where \( x' \) is

\[ x' = \sqrt{1 + k_m^2} x \]

\[ -(k_m b_m - k_m c_i - c_i) / \sqrt{1 + k_m^2} \]  

(12)

Now the CT image reconstruction problem can be converted to.

\[ Y = HW \]  

(13)

2.4 Algebraic Reconstruction Techniques

In ART algorithm, a sequence of vectors \( w^{(0)}, w^{(1)}, \ldots \) \( w^{(p)} \) is produced such that the sequence converges to a satisfactory solution of Equation (13).

Let \( L \) denotes the number of given angles while it moves along its path and \( M \) denotes the number of detectors the total number of all projections is written as \( I_{L \times M} \) [7][8]. Using this notation, the traditional ART algorithm can be represented by the iterations:
where \( w^{(0)} \) is arbitrary (often the zero vector)

\[
\begin{align*}
\lambda^{(n)} & = \lambda^{(0)} \left( 1 - \frac{\| H^T \mathbf{w}^{(n)} \|}{\sum_{j=1}^{N} h_j^2} \right) \\
1 & \leq j \leq N, \quad n = 0, 1, ..., \quad i = n \mod I + 1
\end{align*}
\]  

Where \( \lambda^{(n)} \) is a relaxation parameter, \( 0 < \lambda^{(n)} < 2 \). We call a sequence of iterations a cycle. The relaxation parameter \( \lambda^{(n)} \) controls the rate at which the solution \( w^{(n)} \) is updated in a single iterative step, with values closer to 1 making the algorithm to fit the individual measurement better than when using values closer to 0 (or 2). Although this algorithm has a mathematically well-defined limiting behavior, due to computational costs we would tend to run it for only a few cycles.

3. Experiments and Analysis

3.1 Condition of experiments

The phantom used in the experiments presented here is the 2D Shepp-Logan phantom, which is discretized on a 128\times128 square grid. It is shown in Fig. 1. The projection data for the reconstruction are collected by 128 detectors. We consider one type of scanning geometries of the paths of the line integrals, i.e., parallel-beam, as in Fig. 2.

To estimate the results, we use \( \delta \) as the standard of error estimator, which is as follows.

\[
\delta = \sqrt{\frac{\sum_{x=1}^{width} \sum_{y=1}^{width} (f_{\text{org}}(x, y) - f_{\text{rec}}(x, y))^2}{\sum_{x=1}^{width} \sum_{y=1}^{width} f_{\text{org}}(x, y)^2}} ,
\]

where \( f_{\text{org}}(x, y) \) is the value of pixel \((x, y)\) in the original image. \( f_{\text{rec}}(x, y) \) is the value of pixel \((x, y)\) in the reconstructed image.

In the experiments, the RBF network is employed to represent the phantom image, which maps the coordinate of a point into its pixel. In the structure, the standard deviation \( \sigma \) of the Gaussian function and the number of the hidden neurons are two important factors. If a proper neuron number and \( \sigma \) are selected, we can obtain the best reconstructed image.

3.2 Parameters of the RBF network

In the first part, the reconstructed results with the different RBF net parameters are compared in the experiments, and we can select the best parameter set. The main parameters used in the experiments are listed in Table 1, when the same iteration step and the number of the views are adopted.

From the experiments using different parameters we can draw a conclusion that standard deviation is a very important parameter for the quality of reconstructed image. If its value is selected larger than the proper value, the reconstructed image is too smooth to depict the detail of the original image. Contrarily, a too small standard deviation brings the reconstructed image artifacts and noise. So comparing between the different values of the standard deviation, we select 2 as a proper parameter, when the number of the hidden neurons is 256.

The number of hidden neurons is the other important parameter which can affect considerable the quality of the reconstruction. Generally, it has positive effect on the quality of the reconstructed image. It is better selection that the number of the hidden neurons is set to 256.

3.3 Comparison

In second part, the parameters of the RBF network have been estimated, the number of the hidden neurons is 256 (16\times16) and standard deviation is 2.
To prove the superiority of the method, we compare the reconstructed image of the method with RBFNN.

In the experiments, the projection data set is acquired over one turn with different number of views per turn. We selected 15, 30, 45, 60 as the number of views per turn. In order to get the best reconstruction result, we compare the results acquired from the different number (from 1 to 100) of iteration steps. From experiments, we can conclude when the number of iteration step increased, the error decreased. And the effect is more obvious in the previous several iterations. Considering the time complexity of the reconstruction and the quality of reconstruction, we choose 20, 20, 40, 50 as the number of the iteration for the four conditions (including 15 views per turn, 30 views per turn, 45 views per turn, 60 views per turn). Because page limitation only one reconstructed image and error is shown in figure 3.

The results show that the increase of the projection views affects positively the reconstructed results, and under the same condition, the method is better than FBP. To illustrate the conclusion, we demonstrate the results of two methods in the Fig. 4.

4. Conclusion and future works

In this paper, we have shown that to combine RBF network image representation and ART to reconstruct CT image from a small amount of projection data can get better result. The learning process of the RBF network includes the interpolation and smoothing processes. Usually, the number of weights is less than the number of the pixel and computing the weight vector is easier than calculating the CT image vector. By analyzing the time complexity of the proposed method and ART, we find the proposed method can reconstruct CT image more quickly than ART. The results of the experiments show clearly the advantage over the FBP.

There are still some problems need to be investigated further. For example, if standard deviation parameter of RBF network is changed in the process of iteration, the computation complicacy will increase. This problem will be studied in the future work.

5. References


