A consolidated model of self-fulfilling expectations and self-destroying expectations in financial markets

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\textbf{Abstract}

Self-fulfilling expectations, where people's expectations may enable some 'pattern' to arise, and self-destroying expectations, where people's expectations could also induce the arisen 'pattern' to disappear, are two attracting phenomena in financial markets. We hold that these two seemingly conflicting phenomena originally arise from the intertemporal payoff structure of investors and build a consolidated model to systemically explore their underlying mechanisms. Based on individuals' investments, with trend-following and trend-reversing expectation rules, our model exhibits the process in which one expectation rule goes from showing superior performance to being unprofitable, as it is gradually exploited, realized, and taken advantage of. Adding the fundamentalist rule, we find that the fluctuation of fundamentalists' impacts on prices, driven by individuals' real payoffs, is the crucial factor that enables their wished 'pattern' that prices fluctuate around the supposed fundamental value to arise as well as induces this emerging 'pattern' to disappear.

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\section{Introduction}

As a bridge from the past to the future, expectation is an unavoidable and important factor in economic systems. Individuals' expectations guide their actions, which aggregate the macro outcome that they make their best to predict. In turn, the new outcome updates individuals' information, which renews their expectations. Peoples' expectations and the aggregate outcome coevolve, which exhibits the complexity of the system as Arthur (1999) describes: multiple elements adapt or react to the pattern they create by themselves. During this process, we focus on two fundamental questions: how does the aggregate outcome, guided by people's expectations, evolve? Additionally, if we simplify individuals' diverse expectations as different expectation rules, how do these different rules evolve among each other and over time?

To answer the above questions, two important factors should not be ignored: strategy, that is, how people base their actions from their expectations, and learning, that is, how people renew their expectation rules according to the aggregate realizations. People make strategies according to the economic environment they are involved in and take actions based on strategies and expectations. Their aggregate actions form demand and supply and build the relationship between the average expectation and the realized aggregate outcome, that is, the static inner structure of the system. People's learning determines the way in which they adapt to the system that they create by themselves, that is, the dynamic evolution of the system. In economic systems, the underlying element to determine the two factors is people's payoff. For individuals,
maximizing payoffs is the basis of making decisions and the motivation of updating expectation rules. In short, the payoff structure is the key.

If people’s payoff only contains the information of one period, such as the difference between the realized aggregate outcome and individual’s expectation on it, the two questions are relatively clear and can be classified as two cases according to different strategic environments. There are two basic environments: strategic complementary environment, where an individual has the incentive to act similarly with others, and strategic substitutability environment, where an individual has the incentive to act differently from others. From the aspect of the relationship between people’s average expectation and the aggregate outcome, the related conceptions are positive expectation feedback and negative expectation feedback, respectively. Corresponding to the two feedbacks, Heemeeijer et al. (2009) designed two experimental market environments, with the other settings equivalent, to investigate how different economic environments affect the aggregate market outcome as well as individual forecasting behavior. They find that the market with negative expectation feedback quickly converges, whereas the market with positive expectation feedback converges very slowly (or oscillates) and exhibits significant excess volatility. Meanwhile, the coordination of individuals’ expectation rules is extremely quick in the market with positive expectation feedback, whereas heterogeneity in individuals’ expectations persists (until the market converges to its steady state) in the negative expectation feedback environment. Their conclusions are consistent with the early study, proposed by Fehr and Tyran (2008), about the impacts of different strategic environments (strategic complementary and strategic substitutability) on individual rationality and aggregate outcomes.

However, the financial market is more complicated. On the one hand, if many individuals expect the price of an asset to rise (fall) and therefore start buying (selling) it, the aggregate demand will increase (decrease) and so will the asset price. Bubbles and crashes could arise simply due to people’s expectations. Also, excess volatility is observed as one stylized fact (Cont, 2001). It thus seems that financial market belongs to strategic complementary environment (positive expectation feedback). On the other hand, the market also has the property of strategic substitutability environment (negative expectation feedback): there is no common expectation rule that could bring money to all the traders. The coordination of individuals’ expectations and actions can erode profits and yield losses. Any coordination of individuals’ expectations is temporary, as is the price ‘pattern’ implied by it. Bubbles and crashes are always turning. Most previous anomalies that contain arbitrage opportunities, such as the small-firm effect, and the holiday effect, among others, disappear after they are published. Strong evidence is provided by Wessel et al. (2006), who attribute the self-destruction property of anomalies to more and more practitioners’ anticipating and implementing strategies to take advantage of them. Timmermann (2008) also expresses a similar idea, which is not only limited to the anomalies, but also refers to the predictability of stock market returns by some variables such as the inflation rate as well as the superior performance of some forecasting models. Moreover, Timmermann introduce a life cycle pattern called ‘creative self-destruction’ to describe the process in which a particular forecasting method goes from being seldom realized to being gradually and widely found and adopted to being incorporated into prices and disappearing. As a result, coordination (if it arises), which could dispel individuals’ payoffs, is just one state in the evolution of people’s expectations and individuals need heterogeneity to earn money.

In our opinion, the two seemingly conflicting phenomena in financial markets are due to people’s more complex payoff structure. People invest in one stock because they believe it could bring money to them and increase their wealth by buying at a low price and selling at a high price. Therefore, the payoff structure contains two actions and is intertemporal. On the one hand, if an individual expects the price will rise, he/she opens or holds a long position; otherwise, he/she opens or holds a short position. That is, people take actions following their expectations. Prices increase with the excess demand, which is guided by the majority’s expectation. On the other hand, there is a time lag between an individual’s expectation and his/her action. When individuals make decisions to buy or sell, they just have the information up to the last period and do not know at which price they could buy or sell the stock. In other words, their expectations for the price trend of the next period determine their actions in the current period. They want prices to evolve as they desire after they take their actions, but the coordination of their expectations could lead their wish to come true in advance, which induces them to buy (sell) at a high (low) price and decreases the profitability of the prevailing expectation rule. Therefore, the coevolution of different expectation rules and prices becomes more complicated. Prices could evolve as the majority do, which provides an opportunity for the ‘pattern’ implied in one expectation rule to show up if it is becoming attractive and adopted by more and more people. When the ‘pattern’ shows up, which hints that the information provided by the prevailing expectation rule has been appropriately incorporated into prices, the profitability of this rule decreases until it is negative due to the consistent actions of its supporters, which further destroys the ‘pattern’. In short, due to people’s expectations, one ‘pattern’ implied in a certain expectation rule could arise, and this ‘pattern’ could also be destroyed by being explored and taken advantage of. In this paper, we name the two phenomena ‘self-fulfilling expectations’ and ‘self-destructing expectations’ and build a simple multi-agent model to present the two coexisting phenomena and systematically explore their underlying mechanisms.

An artificial financial market is constructed based on individuals’ investment process. One investment contains a pair of actions: buy and sell. Agents choose the expectation rules, observe the market, take actions, and evaluate the rules in use. Their aggregate actions form the excess demand, which guides the movement of prices, and their aggregate selections of different expectation rules form their distribution. Driven by individuals’ realized payoffs, prices and different expectation rules coevolve in the system. Under this structure, with trend-following and trend-reversing expectation rules, we roughly exhibits the life cycle, as Timmermann mentioned, that both of the rules experience the process from bringing money to adopters to leading them to big losses, going through the process from being seldom realized to being widely found and employed. Adding the fundamentalist rule, the emergence of the ‘pattern’ in which prices fluctuate around a supposed
fundamental value and its subsequent disappearance are both clearly shown in the market. We find the fluctuation of their impacts on prices, driven by individuals’ payoffs, is the crucial factor.

The paper is organized as follows: Section 2 presents the structure of the multi-agent model, Section 3 exhibits the numerical simulation results of two examples with simple expectation rules, and Section 4 gives a brief conclusion.

2. The model

Let us consider a simple stock market populated by \( N \) agents, where only one stock is traded, and for simplicity, we ignore the bid-offer spread to make all the agents able to trade at the same price. For each period \( t \), agents predict the next price trend \( p_{t+1} - p_t \) by some simple expectation rules chosen from a common set \( \{H^1, H^2, ..., H^k\} \) and act according to their expectations and positions in the following way:

\[
  a^i_t = \begin{cases} 
    1 & \text{if } E^i_t(p_{t+1} - p_t) > 0 \text{ and } A^i_{t-1} = 0 \text{ or } 1 \\
    -1 & \text{if } E^i_t(p_{t+1} - p_t) < 0 \text{ and } A^i_{t-1} = 0 \text{ or } 1 \\
    0 & \text{else}
  \end{cases}
\]

where \( a^i_t \) is agent \( i \)'s action at period \( t \), \( A^i_{t-1} = \sum_{k=1}^{t-1} a^i_k \) records agent \( i \)'s position up to period \( t - 1 \), and \( E^i_t(p_{t+1} - p_t) \) represents agent \( i \)'s expectation of the next price trend at period \( t \). Here, for simplicity, we let agents predict the next price trend, which is the qualitatively decisive factor of the payoffs of their investments, instead of employing the accurate price; as for earning money, people care more about the fact that price will rise (fall) after they buy (sell), not the price at which they buy (sell). Agents make their expectations by separately choosing an expectation rule from the common set \( \{H^1, H^2, ..., H^k\} \), which are formed by some simple rules according to the available information up to period \( t - 1 \), and \( h^i_t \in \{1, 2, ..., k\} \) records the number of the expectation rule that agent \( i \) uses at period \( t \). After agents form their expectations, each of them is allowed to trade one share of stock if the last position he/she holds is opposite (to his/her current decision) or empty. That is, as one investment, a buyer (seller) will keep a long (short) position until he/she predicts that the price trend is going to turn over. Then \( A^i_{t-1} \) is bounded in \( \{-1, 0, 1\} \). Additionally, for the traders in our model, observing the market to wait for the arrival of new trading signals (denoted by \( a^i_t = 0 \)) is also an important behavior similar to buying and selling.

After all the agents make their choices, the total excess demand \( D^{ex} = \sum_{i=1}^N a^i_t \) is calculated. Price is assumed to change linearly with \( D^{ex} \) and to be disturbed by noise traders or some exogenous factors, which are included in an i.i.d Gaussian noise \( \epsilon_t \sim N(0, \sigma^2) \):

\[
  p_t - p_{t-1} = \frac{\lambda D^{ex}}{N} + \epsilon_t
\]

where \( \lambda \) reflects the degree to which the excess demand affects the price movement. When price \( p_t \) is announced, the available history information is updated. If agent \( i \) closes his/her position at period \( t \) denoted by \( A^i_t = 0 \), which hints that one
investment ends up, the real payoff of this investment $U_{t}^{i,h_{t}}$ by using the expectation rule $j = h_{t}^{i}$ can be calculated:

$$U_{t}^{i,h_{t}} = -(a_{t}^{i}p_{t} + a_{t-m_{t}}^{i}p_{t-m_{t}})$$

where $m_{t}^{i}$ records the duration of this investment process for agent $i$. Considering that it is probably hard and costly (sometimes impossible) for agents to keep track of all the rules to know which one is better than others and transfer to the one showing the best performance, we make the following simple assumption. If $U_{t}^{i,h_{t}} > 0$, which means the expectation rule $j$ that agent $i$ uses for this investment brings positive profits to him/her, agent $i$ will stick to this expectation rule for the next investment; that is, $h_{t+1}^{i} = h_{t}^{i}$. Otherwise, if $U_{t}^{i,h_{t}} \leq 0$, agent $i$ will re-choose one expectation rule from the common set with the same probability, namely,

$$\text{Prob}\{h_{t+1}^{i} = j'\} = \frac{1}{k} \quad (j' \in \{1, 2, ..., k\})$$

Notice that even if the current rule brings a non-positive profit to agent $i$, he/she also has some probability of keeping it with the aleatory thought that the latest unprofitable rule may have a favorable application in the next investment.
When $A_i^t \neq 0$, which means that agent $i$ still has a position in the investment, he/she will continue to watch the market following the current expectation rule $j$ ($h_i^{t+1} = h_i^j$), and the default value for the payoff of an unclosed investment $U_i^{j,h_i^j}$ is set to 0.

Then, the model is closed. Taking one investment as the unit, some timing properties of the investment process in real life are naturally incorporated into our model. First, the investment durations and timings of updating rules are different for agents using different expectation rules. Second, even for agents using the same rule, their investment timings could also be different by the timings when they adopt the rule.

Fig. 4. Profits of individuals 400–600 at period 2850, 2854 and 2857.
3. Results of numerical simulations for some specific expectation rules

Under the above structure, we explore the phenomena of ‘self-fulfilling expectations’ and ‘self-destroying expectations’ by some simple rules. All of the following results are run over 5000 steps with the same parameters: \( N = 1000, \lambda = 0.3, \sigma = 0.045 \) and initial values: \( p_0 = 0.6, p_1 = 1.1 \).

3.1. Trend-following vs. trend-reversing

The two simplest expectation rules regarding price trends are trend-following (denoted by \( H^1 \)) and trend-reversing (denoted by \( H^2 \)). As their names imply, the former believes that the latest popular trend will go on for some time, and the latter one, which is the total opposite, holds that no trend will last and expects the price trend to reverse. Then the common set of expectation rules is specified as \( \{ H^1, H^2 \} \), and for each period, the two expectation rules separately give the following prediction:

\[
H^1(p_{t+1} - p_t) = p_{t-1} - p_{t-2} \\
H^2(p_{t+1} - p_t) = p_{t-2} - p_{t-1}
\]
Without loss of generality, we give the same proportion of users to the two expectation rules at the beginning and get the following general results.

Along with the evolution of prices, the proportions of the two expectation rules fluctuate up and down 0.5, as shown in Fig. 1 (here we just exhibit the fluctuations of proportion of trend followers \( n^1_t \)), and the proportion of trend reversers equaling \( 1 - n^1_t \) has a similar fluctuation feature. It means that neither of the two expectation rules could beat the other one and maintain priority forever. Coordination on one expectation rule is impossible. A closer look is shown in Fig. 2, from which we can clearly see that during the evolving process, trend followers and trend reversers usually alternatively take the dominant position, while there are also few intermittent interruptions in which either of them could remain lucky for a longer period of time.

Recalling the way in which agents change their expectation rules, this result means that the minority wins most of time with the feature that the proportion of the expectation rule they choose will rise in the next investment. Conversely, the majority loses most of time, with the feature that the corresponding rule loses its supporters in the following investment. This result is reasonable and in accordance with the reality. People get a positive payoff if the price rises after they buy and falls after they sell. However, their consistent actions could make themselves buy at a high price and sell at a low price, which
reduces the probability of their earning money. We hypothesize this is also the basic point from which many econophysicists simplify financial markets as minority games (Challet and Zhang, 1997, 1998; Challet et al., 2001, 2005). On the other hand, since Keynes (1936) described financial markets as a beauty contest, many studies, especially those on bubbles and crashes, often take the markets as majority games. Debates on this subject are still active (Marsili, 2001; Andersen and Sornette, 2003). In our opinion, both of them are correct but from different aspects. The former one is from the angle of people’s real payoff in one investment, as we do in this paper, whereas the latter one is from the angle of investors’ virtual wealth, which increases (decreases) along with asset prices rising (falling). Considering diverse random factors in reality, in our model, we also show some situations that the ‘majority’ wins with its priority position strengthened by an increased proportion in the following investment. These arise from the random noise we put in the price formula, and the frequency of majority's winnings is affected by the size of the noise. The following microanalyses give us a more vivid impression.

Fig. 3 takes period 2846–2862 as an example, during which three consecutive investment processes are marked by rectangles, and the inside upward and downward arrows separately denote buy and sell behaviors of trend followers. It is easy to see that trend followers earn money in the first two investments when they buy the stock at a lower price and sell it at a higher price, but for the last investment with above 80 percent trend followers in the market, they have big losses. A closer look at the payoffs of individuals (take agents 400–600 as an example) during the three investments is shown in Fig. 4, from which we can see that the more attractive an expectation rule (trend-following in this example) is, the less it can bring profits to each individual. The natural feature of pursuing profits makes agents try other rules when they face losses, but their actions could dispel the profitability of the previous better rule, eventually bringing losses to both themselves and earlier users by their aggregate effect on prices.

Under the simple assumption about agents’ way of updating expectation rules, this example with two simplest expectation rules shows us the general idea of one rule’s life cycle, as Timmermann (2008) mentioned, that it goes from being profitable with few users to becoming more and more unprofitable with more and more traders detecting and taking advantage of it, although the duration of this process may be different in the way in which people update rules and how easily a rule with good performance can be detected.

In this simple example, it is not obvious to see the ‘pattern’ implied in one expectation rule arise and subsequently disappear in the evolution of prices, as the two expectation rules we use here are so sensitive to noises that their ‘patterns’ are too short to be distinct. Additionally, the timing properties of agents’ investments are not shown here for the reason that the two expectation rules are opposite to one another, with the same timing for agents to trade and evaluate. Meanwhile, for the same rule, the early supporters just finish one investment when the new join in, which makes the timings of their following actions the same, as are their payoffs.

3.2. Fundamentalist vs. trend-following and trend-reversing

Based on the former example, we add the fundamentalist rule, which holds different investment timings from trend-following and trend-reversing and implies a more distinct ‘pattern’ that prices fluctuate around a supposed fundamental value. Then, the following questions are focused.

(i) Could fundamentalists drive out other traders to maintain a priority in the market?
(ii) Does the ‘pattern’ that fundamentalists desire, where prices fluctuate around the supposed fundamental value, arise?
(iii) Does the emergent ‘pattern’ disappear along with fundamentalists detecting and taking advantage of it?
Fundamentalist (denoted by $H^3$) believes that there exists a fundamental value that is the reflection of real economic factors and magnetizes prices to fluctuate around it. Therefore, the fundamentalist predicts that the price will rise if it is below the fundamental value and fall if it is above it, specified as

$$H^3(p_{t+1} - p_t) = p_t - p_{t-1}$$

The common set of expectation rules is now $\{H^1, H^2, H^3\}$. Without loss of generality, we set the supposed fundamental value $p_t$ equal to 1 and give the same proportion of users to the three expectation rules at the beginning. The general results are as follows.

Fig. 5 shows the evolution of prices in the left, from which it is easy to see that prices fluctuate around the supposed fundamental value of 1 do arise as fundamentalists wish, but that this is occasionally interrupted by some temporary
deviations as well as large bubbles and crashes. Along with the evolution of prices, fundamentalists also could not drive out all the other traders forever, as shown in the right graph. Comparing the two graphs, corresponding states between the evolution of prices and the fluctuation of different expectation rules are found. When prices fluctuate around their supposed fundamental value of 1, the fundamentalists’ proportions fluctuate almost below 0.5; when prices temporarily deviate from 1, the fundamentalists’ proportions increase for a while. A closer look is exhibited in Fig. 6.

Taking one investment as the unit, during one deviation, one fundamentalist only contributes one action to the price movement and then observes the market and waits for the price to touch the supposed fundamental value again. Although fundamentalists’ strength in pulling price back to their supposed value is accumulated by their increasing proportion, for each step, their impact on price may be limited comparing with the impacts of other expectation holders as well as the random noise. Some situations in which the deviation still exists, after the fundamentalist rule has, little by little, attracted most of traders (i.e., the proportion of fundamentalists is close to 1), could arise, then the evolution of prices is mostly affected by random noise. During these periods, a greater level of random noise indicates a higher instability of the evolution of prices. To show the two corresponding states clearly, the simulation results we analyze here adopt a relatively big noise ($σ = 0.045$), which induces some extreme situations that prices could not be driven to the supposed fundamental value, even all the traders become fundamentalists and then random bubbles or crashes driven by noises arise. With a small noise ($σ = 0.005$), we also get the similar qualitative results but with small deviations, as shown in Figs. 7 and 8.

Apparently, fundamentalists’ self-fulfilling and self-destroying expectations coexist in the simple artificial financial market. Then, the following questions arise:

(i) How does the ‘pattern’ wherein prices fluctuate around the supposed fundamental value arise from deviations as fundamentalists wish?
(ii) How does the emergent ‘pattern’ disappear for some time, followed by the reappearance of deviations?

With macro and micro analyses, the two questions are explored in detail in the following. According to different representations in the evolution of prices along with fluctuations of fundamentalists’ proportions, some characteristic segments in Fig. 6 as well as the corresponding micro individuals’ investment processes and payoffs (also take agents 400–600, for example) are expatiated in Figs. 9–14. Rectangles frame fundamentalists’ investment processes with upward and downward arrows separately representing fundamentalists’ buy and sell actions. Double arrows roughly show the difference in fundamentalists’ impacts on price between the end of one investment and the beginning of the next investment by the difference in the multiplication of price change and fundamentalists’ action.

Specifically speaking, deviations from the supposed fundamental value provide opportunities for fundamentalist, attracting its supporters due to its longer investment duration and later evaluation timing than other expectation rules. While fundamentalists wait for the price to return to their supposed fundamental value, trend followers and trend reversers keep trading, evaluating, and trying other expectation rules. Losses incurred by either group could expand fundamentalists; that is, the proportion of fundamentalists would continue to rise, and their accumulated strength will continue to pull the price until it is back to the fundamental value. In brief, the opportunity for fundamentalists to accumulate their impacts on prices provides the precondition for the emergence of their ‘pattern’.

When price touches the supposed fundamental value again, all the fundamentalists accumulated in this deviation period close their positions by the same action at the same time, which probably pushes the price away from the fundamental value in the opposite direction. Then all the fundamentalists calculate their payoffs for evaluation. Usually, both winners and losers
exist depending on when they invest in the market (as shown in Fig. 10). The fact is that if they join the prevailing group later on, they face a higher probability of incurring losses. Additionally, some of the losers will change their expectation rules, which makes the proportion of fundamentalists decrease.

The important factor used to determine whether fundamentalist's 'pattern' shows up or not and how long it could remain is the difference in the fundamentalists' impacts on prices between the end of one investment and the beginning of the following investment. Specifically, fluctuations in fundamentalists' proportions, the interactions between different expectation rules (for instance, coincident actions of different expectation rules) and random noises matter.

If the difference is great, which implies that at the beginning of the next investment, fundamentalists have limited power to pull the price in their favorite direction, the price probably continues floating beyond the supposed fundamental value for a while until their accumulated strength is powerful enough to pull it back. This is how the long-period deviation, along with several repeating cycles (i.e., accumulated increase to decrease and vice versa) in fundamentalists' proportions, emerges during periods 700–950 (as shown in Fig. 6). Whereas if the difference is small, which implies that fundamentalists could pull prices back close to their desired line in the following investment, the supposed 'pattern', that prices fluctuate around
Fig. 13. Time series of prices and proportions of fundamentalists during periods 1140–1178.

Fig. 14. Profits of individuals 400–600 at period 1158 and 1175. Note: some figs of profits of individuals at a certain period only show the profits of fundamentalists because the other traders do not close their positions and the default profit for an unfinished investment is set to 0.
the fundamental value, probably emerges. An example is shown in Fig. 9, where fundamentalists’ supposed ‘pattern’ shows up after temporary deviations. Individuals’ corresponding payoffs of three consecutive investments are exhibited in Fig. 10. Along with great losses faced by all the fundamentalists in period 974, a large decrease in fundamentalists’ proportion induces a large decrease in their impact on price, and a long-period deviation follows. During the deviation, fundamentalists keep pulling prices back, with their proportions consequentially increasing. When price meets their desired line in period 990, all of the fundamentalists accumulated in this deviation close their positions in the next period. With a minority of winners and a majority of losers in period 991, fundamentalist loses few supporters in the next investment again, and its impact on price also decreases in some degree. But this time, the difference is smaller than the last one and a shorter deviation follows. After the short deviation, with a majority of winners, the proportion of fundamentalists increases a little, as does their impact on price. Consequently, fundamentalists’ desired ‘pattern’ emerges in the following periods.

Along with the ‘pattern’ that prices frequently fluctuate around the supposed fundamental value, fundamentalists frequently trade and evaluate their expectation rule, as trend followers and trend reversers do. They cannot always win from following their wished ‘pattern’, and their proportions, formed by different members, always fluctuate almost below 0.5. An example is shown in Figs. 11 and 12, from which we can see along with the ‘pattern’, fundamentalists, composed by flexible ‘minority’ members, frequently trade and face different and uncertain payoffs.

In fact, during the ‘pattern’ period, with prices frequently above and below the supposed fundamental value, fundamentalists’ more consistent actions induced by their frequent trading have two sides. From one side, they are in favor of leading other expectation holders to timely pull prices back to their desired line; from the other side, they could dispel fundamentalists’ profits and keep their proportion low (almost below 0.5), which makes it easier for fundamentalists to lose control of prices after they face large losses. An accidental large loss could bring a big decrease to their proportion as well as their impact on price. When it happens, fundamentalists’ ‘pattern’ disappears (at least somehow), and deviations arise along with a new run of accumulation in their proportion. For example, a deviation appears following fundamentalists’ overall losses in period 1158, which induces a large decrease in both their proportion and their impact on price; this event is shown in Figs. 13 and 14. In brief, fundamentalists’ supposed ‘pattern’ is destroyed due to the coordination of their expectations and actions, which induces sudden overall losses and decreases their impacts on prices.

In conclusion, with fundamentalists joining in the market, the fluctuation in their impacts on prices is important for their self-fulfilling and self-destroying expectations. The accumulation in their impacts on prices provides the opportunity for prices to evolve as they desire. However, the coordination of their expectations and actions decreases the probability of their desired ‘pattern’ prevailing by dispelling their profits of insisting on the fundamentalist rule. A sudden overall loss could induce a sudden large decrease in their impact on prices, which causes the previous ‘pattern’ to disappear. During the ‘pattern’ periods, the information provided by the fundamentalist rule is appropriately incorporated into prices, and fundamentalists, composed of different members, face different and uncertain payoffs.

4. Conclusions

As a distinct feature from natural systems, expectation adds a layer of complexity to social systems. How the aggregate outcome, guided by people’s expectations, evolves and, with new realized outcomes continuously arriving, how people’s expectations evolve are two natural questions and have been attracting more and more studies (Hommes and Wagener, 2009; Anufriev and Hommes, 2009; Heemeyer et al., 2009; Fehr and Tyran, 2008; Brock and Hommes, 1998). Strategic environment has been found important in their coevolution (Heemeyer et al., 2009; Fehr and Tyran, 2008). However, for financial markets, strategic environment is not enough to explain some phenomena emerging in their coevolution, such as ‘self-fulfilling expectations’ and ‘self-destroying expectations’, which are the focus of this paper. We propose that individuals’ payoff structure also plays an important role in their coevolution. Originating from this idea, a multi-agent model is constructed based on individuals’ investment process. With trend-following and trend-reversing expectation rules, we roughly show the life cycle of an expectation rule as described by Timmermann (2008). It comes from individuals’ real payoffs, and we consider it is the underlying feature of the phenomena of ‘self-fulfilling expectations’ and ‘self-destroying expectations’. Furthermore, we add fundamentalist rule in the market and clearly exhibit the cycle that fundamentalists’ desired ‘pattern’ (i.e., prices fluctuate around the supposed fundamental value) gradually shows up, prevails and then disappears. During this process, we find that the fluctuation of fundamentalists’ impacts on prices, which is driven by individuals’ real payoffs, is crucial for the self-fulfillment and self-destruction of their expectations. Moreover, we also find when the fundamentalist’s ‘pattern’ prevails, which implies that the information provided by the fundamentalist rule is appropriately incorporated into prices, fundamentalists, composed of different members, face different and uncertain payoffs. From the perspective of the payoff structure, we hope that our study can make some contributions to the research on the coevolution of prices and different forecasting rules in financial markets and bring some new insights to the debates on the existence of forecasting rules with good performances and the efficiency of the financial markets (Timmermann, 2008; Timmermann and Granger, 2004; Tvede, 2002; Schmidt, 2002; Lo et al., 2000; Zhang, 1999; Menkhoff, 1997; Campbell et al., 1997).

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